KPK BOARD NOTES

MATHEMATICS

9TH CLASS

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Urdu Books Whatsapp Group

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Chapter # 2

Ex # 2.1

Set of Natural Numbers

$$N = \{1, 2, 3, 4, \ldots\}$$

Set of Whole Numbers

$$W = \{0, 1, 2, 3, 4, \ldots\}$$

Set of Integers

$$Z = \{0, \pm1, \pm2, \pm3, \ldots\}$$

OR

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3\}$$

Rational Numbers

The word Rational means "Ratio".

A rational number is a number that can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. Rational numbers is denoted by Q.

Set of Rational Numbers

$$Q = \left\{\frac{p}{q} \mid p, q \in Z, q \neq 0\right\}$$

Irrational Numbers

The word Irrational means "Not Ratio".

Irrational number consists of all those numbers which are not rational. Irrational numbers is denoted by $Q^{/}$.

Real numbers

The set of rational and irrational numbers is called Real Numbers. Real numbers is denoted by R.

Thus $Q \cup Q^{/} = R$

Note:

All the numbers on the number line are real numbers.

Terminating Decimal Fraction:

A decimal number that contains a finite number of digits after the decimal point.

Non-Terminating Decimal Fraction:

A decimal number that has no end after the decimal point.

Non-Terminating Repeating Decimal Fraction

In non-terminating decimal fraction, some digits are repeated in same order after decimal point.

Non-Terminating Non-Repeating Decimal Fraction.

In non-terminating decimal fraction, the digits are not repeated in same order after decimal point.

Decimal Representation of Rational and Irrational Numbers.

(i) All terminating and repeating decimals are rational numbers.

(ii) Non-terminating recurring (repeating) decimals are rational numbers.

(iii) Never terminating or repeating decimals are irrational numbers.

Non-terminating and non-recurring (repeating) decimals are irrational numbers.

Note:

(i) Repeating decimals are called recurring decimals.

(ii) Non-repeating decimals are called non-recurring decimals.

Chapter # 2

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In Questions 1 – 10, consider the numbers.

$$\mathbf{2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}, 0.333 \ldots}$$

1. Which are whole numbers?

Ans: $3, \quad 0, \quad \sqrt{36}, \quad 1$

2. Which are integers?

Ans: $3, \quad 0, \quad \sqrt{36}, \quad -9, \quad 1$

3. Which are irrational numbers?

Ans: $\sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi$

4. Which are natural numbers?

Ans: $3, \quad \sqrt{36}, \quad 1$

5. Which are rational numbers?

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, -9, 1, \quad 4\frac{2}{3},$ $0.333 \ldots$

6. Which are real numbers?

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}$ $, 0.333.$

7. Which are rational numbers but not integers?

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad 4\frac{2}{3}, \quad 0.333 \ldots$

8. Which are integers but not whole numbers?

Ans: -9

9. Which are integers but not natural numbers?

Ans: $0, -9$

10. Which are real numbers but not integers?

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad \sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi,$ $4\frac{2}{3}, \quad 0.333 \ldots$

11. Write the decimal representation of each of the following numbers.

$\frac{1}{6}, \frac{6}{7}, \frac{2}{9}, \frac{1}{8}$

$\frac{1}{6} = 0.1666 \ldots.$

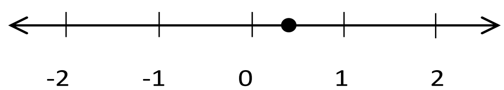
$\frac{6}{7} = 0.8571 \ldots.$

$\frac{2}{9} = 0.222 \ldots.$

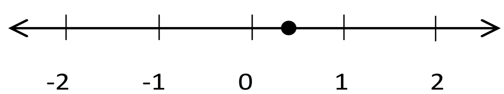
$\frac{1}{8} = 0.125$

12 Depict each number on a number line.

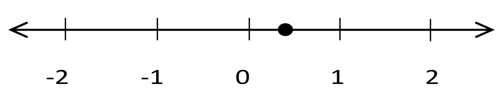
(i) $\mathbf{\frac{1}{3} = 0.333 \ldots}$



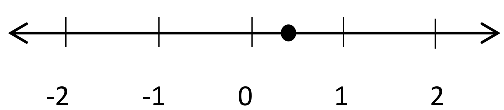
(ii) $\mathbf{\frac{1}{4} = 0.25}$



(ii) $\frac{1}{9} = 0.111 \ldots$



(iv) $\frac{1}{10} = 0.1$



ختم نبوت ﷺ زندہ باد　　　　عظمت صحابہ زندہ باد

السلام علیکم ورحمۃ اللہ وبرکاتہ:

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈ من "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

- ❖ گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈ من کی اجازت کے کسی بھی قسم کی (اسلامی و غیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔
- ❖ گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
- ❖ کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔
- ❖ ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- ❖ اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈ من سے رابطہ کیجئے۔
- ❖ سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی و ذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- ❖ تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- ❖ عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- ❖ **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- ❖ اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈ من سے وٹس ایپ پر بذریعہ میسج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

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اللہ تبارک تعالیٰ ہم سب کا حامی و ناصر ہو

Ex # 2.2

Properties of Real Number

The set R of real number is the union of two disjoint sets. Thus $R = Q \cup Q^{/}$

Note:

$$Q \cap Q^{/} = \emptyset$$

Real Number System

Closure Property w.r.t Addition

The sum of real number is also a real number.

If $a,\ b\ \in R$ then $a + b\ \in R$

Example:

$$7 + 9 = 16$$

Where 16 is a real number.

Closure Property w.r.t Multiplication

The Product of real number is also a real number.

If $a,\ b\ \in R$ then $a\,.\,b\ \in R$

Example:

$$7 \times 9 = 63$$

Where 63 is a real number.

Commutative Property w.r.t Addition

If $a,\ b\ \in R$ then $a + b = b + a$

Example:

$$7 + 9 = 9 + 7$$
$$16 = 16$$

Commutative Property w.r.t Multiplication

If $a,\ b\ \in R$ then $a\,.\ b = b\ .\ a$

Example:

$$7 \times 9 = 9 \times 7$$
$$63 = 63$$

Associative Property w.r.t Addition

If $a,\ b,\ c\ \in R$ then

$$a + (b + c) = (a + b) + c$$

Example:

$$2 + (3 + 5) = (2 + 3) + 5$$
$$2 + 8 = 5 + 5$$
$$10 = 10$$

Associative Property w.r.t Multiplication

If $a,\ b,\ c\ \in R$ then

$$a(bc) = (ab)c$$

Example:

$$2(3 \times 5) = (2 \times 3)5$$
$$2(15) = (6)5$$
$$30 = 30$$

Additive Identity

Zero (0) is called Additive identity because adding "0" to a number does not change that number.

If $a \in R$ there exists $O\ \in R$ then

$$a + 0 = 0 + a = a$$

Example:

$$3 + 0 = 0 + 3 = 3$$

Multiplicative Identity

1 is called Multiplicative identity because multiplying "1" to a number does not change that number.

If $a \in R$ there exists $1\ \in R$ then

$$a\,.\,1 = 1\,.\,a = a$$

Example:

$$3 \times 1 = 1 \times 3 = 3$$

Additive Inverse

When the sum of two numbers is zero (0)

If $a \in R$ there exists an element $a^{/}$ then $a + a^{/} = a^{/} + a = 0$ then $a^{/}$ is called additive inverse of a

Or

$$a + (-a) = -a + a = 0$$

Example:

$$3 + (-3) = 3 - 3 = 0$$
$$-3 + 3 = 0$$

Ex # 2.2

Multiplicative Inverse

When the Product of two numbers is "1".

If $a \in R$ and $a \neq 0$ there exists an element $a^{-1} \in R$ then

$a\,.\,a^{-1} = a^{-1}\,.\,a = 1$ then a^{-1} is called multiplicative inverse of a

Or

$$a\,.\,\frac{1}{a} = \frac{1}{a}.a = 1$$

Example:

$$3 \times \frac{1}{3} = \frac{1}{3} \times 3 = 1$$

Distributive Property of Multiplication over Addition

If $a,\ b,\ c\ \in R$ then

$$a(b+c) = ab + ac$$
$$(b+c)a = ba + ca$$

Example:

$$2(3+5) = 2 \times 3 + 2 \times 5$$
$$2(8) = 6 + 10$$
$$16 = 16$$

Properties of Equality of Real Numbers

Reflexive Property of equality

Every number is equal to itself.

$$a = a$$

Example:

$$3 = 3$$

Symmetric Property of Equality

If $a = b$ then also $b = a$

Examples:

$$x = 5$$
$$or\ 5 = x$$
$$x^2 = y$$
$$or\ y = x^2$$

Transitive Property of Equality

If $a = b$ and $b = c$ then $a = c$

Example:

$if\ x + y = z$ and $z = a + b$

Then $x + y = a + b$

Ex # 2.2

Additive Property of Equality

If $a = b$ then also $a + c = b + c$

Examples:

$$x - 3 = 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

$$x + 3 = 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

Multiplicative Property of Equality

If $a = b$ then also $a.\ c = b\ .\ c$

Or

$a = b$ then $\frac{a}{c} = \frac{b}{c}$

Examples:

$$\frac{x}{3} = 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

$$2x = 24$$

Divide B.S by 2

$$\frac{2x}{2} = \frac{24}{2}$$
$$x = 12$$

Cancellation Property w.r.t Addition

If $a + c = b + c$ then $a = b$

Examples:

$$2x + 5 = y + 5$$
$$2x = y$$
$$2x - 5 = y - 5$$
$$2x = y$$

Chapter # 2

Ex # 2.2

Cancellation Property w.r.t Multiplication

If $a\,.\,c = b\,.\,c$ then $a = b$

OR

If $\dfrac{a}{c} = \dfrac{b}{c}$ then $a = b$

Examples:

$$2x \times 5 = y \times 5$$
$$2x = y$$
$$\frac{2x}{5} = \frac{y}{5}$$
$$2x = y$$

Properties of Inequality of Real Numbers

Trichotomy Property

Trichotomy property means when comparing two numbers, one of the following must be true:

$$a = b$$
$$a < b$$
$$a > b$$

Examples:

$$5 = 5$$
$$3 < 5$$
$$3 > 5$$

Transitive Property

(i) **If $a > b$ and $b > c$ then $a > c$**

Example:

If $7 > 5$ and $5 > 3$ then $\mathbf{7 > 3}$

(ii) **If $a < b$ and $b < c$ then $a < c$**

Example:

If $3 < 5$ and $5 < 7$ then $\mathbf{3 < 7}$

Additive Property

(i) **If $a < b$ then $a + c < b + c$**

Example:

$3 < 5$ then $3 + 2 < 5 + 2$

$$x - 3 > 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

Ex # 2.2

(ii) **If $a > b$ then $a + c > b + c$**

Example:

(a) $5 > 3$ then $5 - 2 > 3 - 2$

(b) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$

(c) $$x + 3 > 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

Multiplicative Property

When $c > 0$:

(i) If $a < b$ then $ac < bc$

(ii) If $a > b$ then $ac > bc$

Example:

(a) $5 > 3$ then $5 \times 2 > 3 \times 2$

(b) $$\frac{x}{3} > 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 > 5 \times 3$$
$$x > 15$$

$$2x > 24$$

Divide B.S by 2

$$\frac{2x}{2} > \frac{24}{2}$$
$$x > 12$$

When $c < 0$:

(i) If $a < b$ then $ac > bc$

(ii) If $a > b$ then $ac < bc$

Example:

(a) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$

(b) $$\frac{x}{-3} < 5$$

Multiply B.S by -3

$$\frac{x}{-3} \times -3 > 5 \times -3$$
$$x > -15$$

Example: 4

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Solve the following equation using properties of real numbers.

$\boldsymbol{2x - 5 = 3x + 4}$

Solution:

$2x - 5 = 3x + 4$	
$2x - 5 + 5 = 3x + 4 + 5$	$\therefore\ a = b \text{ then } a + c = b + c$
$2x - 5 + 5 = 3x + 9$	$\therefore$ *Closure Property w.r.t Additon*
$2x + 0 = 3x + 9$	$\therefore\ -5\ \&\ 5$ *are additive inverse*
$2x = 3x + 9$	$\therefore\ 0$ *is the additive identity*
$3x + 9 = 2x$	$\therefore$ *Symmetric Property*
$3x + 9 - 2x = 2x - 2x$	$\therefore\ a = b \text{ then } a - c = b - c$
$3x - 2x + 9 = 0$	$\therefore\ 2x\ \&\ -2x$ *are additive inverse*
$(3 - 2)x + 9 = 0$	$\therefore$ *Distributive Property*
$1.x + 9 = 0$	
$x + 9 = 0$	$\therefore\ 1$ *is Multiplicative Identity*
$x + 9 - 9 = 0 - 9$	$\therefore\ a = b \text{ then } a - c = b - c$
$x + 9 - 9 = -9$	$\therefore\ 0$ *is the Additive Identity*
$x + 0 = -9$	$\therefore\ 9\ \&\ -9$ *are additive inverse*
$x = -9$	$\therefore\ 0$ *is the Additive Identity*

Ex # 2.2
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Q1: Name the properties used in following equations.

(i) $\boldsymbol{1 + (4 + 3) = (1 + 4) + 3}$

Ans: Associative law of addition

(ii) $\boldsymbol{5(a + b) = 5a + 5b}$

Ans: Distributive law of multiplication over addition

(iii) $\boldsymbol{a + 0 = 0 + a = a}$

Ans: Additive identity

(iv) $\boldsymbol{5 \times \frac{1}{5} = \frac{1}{5} \times 5 = 1}$

Ans: Multiplicative inverse

Q2: Write the missing number.

(i) $\boldsymbol{2 + (\_\_\_ + 4) = (2 + 6) + 4}$

Answer: 6

(ii) $\boldsymbol{7 + (4 + 2) = 13}$**, so** $\boldsymbol{(7 + 4) + 2 =}$____

Answer: 13

(iii) $\boldsymbol{9 \times (3 \times 4) = 108}$**, so** $\boldsymbol{(9 \times 3) \times 4 =}$_____

Answer: 108

(iv) $\boldsymbol{5 \times (8 \times 9) = (5 \times \_\_\_\_) \times 9}$

Answer: 8

Q3: Chose the correct option

(i) $\boldsymbol{8 \times (6 \times 7)}$ **is equal to:**

(a) $8 \times 6 - 7$ **(b)** $8 - (6 - 7)$

(c) 8×12 **(d)** $(8 \times 6) \times 7$

Answer: d. $(8 \times 6) \times 7$

(ii) Which one of the following illustrates the Associative Law of Addition?

(a) $3 + (2 + 4) = (4 + 4) + 1$

(b) $3 + (2 + 4) = (3 + 2) + 4$

(c) $3 + (2 + 4) = (5 + 2) + 2$

(d) $3 + (2 + 4) = (2 + 6) + 1$

Answer: b. $3 + (2 + 4) = (3 + 2) + 4$

Ex # 2.2

(iii) Which one of the following illustrates the Associative Law of Multiplication?

(a) $4 \times (3 \times 6) = (6 \times 6) \times 2$

(b) $4 \times (3 \times 6) = (3 \times 12) \times 2$

(c) $4 \times (3 \times 6) = (4 \times 3) \times 6$

(d) $4 \times (3 \times 6) = (3 \times 8) \times 3$

Answer: c. $4 \times (3 \times 6) = (4 \times 3) \times 6$

Q4: Do this without using distributive property.

(i) $\mathbf{39 \times 63 + 39 \times 37}$

Solution:

$39 \times 63 + 39 \times 37$
$= 2457 + 1443$
$= 3900$

(ii) $\mathbf{81 \times 450 + 81 \times 550}$

Solution:

$81 \times 450 + 81 \times 550$
$= 36450 + 44550$
$= 81000$

(iii) $\mathbf{50 \times 161 - 50 \times 81}$

Solution:

$50 \times 161 - 50 \times 81$
$= 8050 - 4050$
$= 4000$

(iv) $\mathbf{827 \times 60 - 327 \times 60}$

Solution:

$827 \times 60 - 327 \times 60$
$= 49620 - 19620$
$= 30000$

Ex # 2.3

RADICALS AND RADICANDS

$\sqrt[n]{a}$ is the radical form of the nth root of a.

$a^{\frac{1}{n}}$ is the exponential form of the nth root of a.

If $n = 2$ then it becomes square root and write $\sqrt{a}$ instead of $\sqrt[2]{a}$

If $n = 3$ then it is called cube root like $\sqrt[3]{a}$

If $n = 5$ then it is called 5th root like $\sqrt[5]{625}$

Important Notes

(i) If a is positive, then the nth root of a is also positive.

Example:

$$\sqrt[3]{64} = \sqrt[3]{(4)^3} = 4$$

(ii) If a is negative, then n must be odd for the nth root of a to be a real number.

Example:

$$\sqrt[3]{-\mathbf{64}} = \sqrt[3]{(-\mathbf{4})^{\mathbf{3}}} = -\mathbf{4}$$

(iii) If a is zero, then $\sqrt[n]{0} = 0$

Properties of Radicals:

Product Rule of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a}\,.\,\sqrt[n]{b}$

Example:

$\sqrt{6x}\sqrt{6y^2}$
$\sqrt{(6x)(6y^2)} = \sqrt{36y^2x} = \sqrt{36}\sqrt{y^2}\sqrt{x}$
$= 6y\sqrt{x}$

$\sqrt{6x}\sqrt{6x^2}$
$\sqrt{(6x)(6x^2)} = \sqrt{36x^2x} = \sqrt{36}\sqrt{x^2}\sqrt{x}$
$= 6y\sqrt{x}$

Ex # 2.3

Quotient Rule of Radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Example:

Simplify: $2\sqrt{\frac{150xy}{3x}}$

Solution:

$$2\sqrt{\frac{150xy}{3x}} = 2\sqrt{50y} = 2\sqrt{5 \times 5 \times 2y}$$

$$= 2\sqrt{5^2}\sqrt{2y} = 2(5)\sqrt{2y} = 10\sqrt{2y}$$

Radical form of an Expression:

The number or quantity that is written under a radical sign ($\sqrt{\ }$ or $\sqrt[n]{\ }$) is called radical form of an expression.

Example:

$\sqrt{9}$ is the radical form of 3.

Exponential form of an Expression:

The number or quantity that is written in the form of exponent is called exponential form of an expression.

Example:

3^2 is the exponential form of 9.

Radical Form	Exponential Form
$\sqrt[n]{a}$	$a^{\frac{1}{n}}$
$\sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$	$a^{\frac{m}{n}}$
$\sqrt[n]{a^n}$	a

Some frequently used radicals are given in the following table

Square Root	Cube Root	Fourth Root
$\sqrt{1} = 1$	$\sqrt[3]{1} = 1$	$\sqrt[4]{1} = 1$
$\sqrt{4} = 2$	$\sqrt[3]{8} = 2$	$\sqrt[4]{16} = 2$
$\sqrt{9} = 3$	$\sqrt[3]{27} = 3$	$\sqrt[4]{81} = 3$
$\sqrt{16} = 4$	$\sqrt[3]{64} = 4$	$\sqrt[4]{256} = 4$
$\sqrt{25} = 5$	$\sqrt[3]{125} = 5$	$\sqrt[4]{625} = 5$
$\sqrt{36} = 6$	$\sqrt[3]{216} = 6$	$\sqrt[4]{1296} = 6$

Example 5 Page # 61

What is the difference between (i) $x^2 = 16$ (ii) $x = \sqrt{16}$?

(i) $x^2 = 16$

Solution:

$$x^2 = 16$$

This means what numbers squared becomes 16. Thus x can be 4 or -4 like $(4)^2 = 16$ and also $(-4)^2 = 16$.

Hence the value of $x = \pm 4$.

(ii) $x = \sqrt{16}$

Solution:

$$x = \sqrt{16}$$

Here x is the principal square root of 16, which has always a positive value such is $x = 4$.

Ex # 2.3

Page # 64

Q1: 64 **Write down the index and radicand for each of the following expressions.**

(i) $\sqrt{\frac{11}{y}}$

$index = 2, radicand = \frac{11}{y}$

(ii) $\sqrt[3]{\frac{13}{3x}}$

$index = 3, radicand = \frac{13}{3x}$

(iii) $\sqrt[5]{ab^2}$

$index = 5, radicand = ab^2$

Q2: 64 **Transform the following radical forms into exponential forms. Do not simplify.**

(i) $\sqrt{36}$

Exponential form= $(36)^{\frac{1}{2}}$

(ii) $\sqrt{1000}$

Exponential form= $(1000)^{\frac{1}{2}}$

(iii) $\sqrt[3]{8}$

Exponential form= $(8)^{\frac{1}{3}}$

(iv) $\sqrt[n]{q}$

Exponential form= $(q)^{\frac{1}{n}}$

(v) $\sqrt{(5-6a^2)^3}$

$((5-6a^2)^3)^{\frac{1}{2}}$

Exponential form= $(5-6a^2)^{\frac{3}{2}}$

(vi) $\sqrt[3]{-64}$

Exponential form= $(-64)^{\frac{1}{3}}$

Ex # 2.3

Q3: 64 **Transform the following exponential form of an expression into radical form.**

(i) $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

(ii) $x^{-\frac{3}{2}}$

$(x^{-3})^{\frac{1}{2}}$

$\sqrt{x^{-3}}$

(iii) $(-8)^{\frac{1}{5}}$

$\sqrt[5]{-8}$

(iv) $y^{\frac{3}{4}}$

$(y^3)^{\frac{1}{4}}$

$\sqrt[4]{y^3}$

(v) $b^{\frac{4}{5}}$

$(b^4)^{\frac{1}{5}}$

$\sqrt[5]{b^4}$

(vi) $(3x)^{\frac{1}{q}}$

$\sqrt[q]{3x}$

Q4: **Simplify:**

(i) $\sqrt[3]{125x}$

Solution:

$\sqrt[3]{125x}$

$= (125x)^{\frac{1}{3}}$

$= (125)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= (5 \times 5 \times 5)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= 5(x)^{\frac{1}{3}}$

$= 5\sqrt[3]{x}$

Ex # 2.3

(ii) $\sqrt[3]{\dfrac{8}{27}}$

$= \left(\dfrac{8}{27}\right)^{\frac{1}{3}}$

$= \left(\dfrac{2 \times 2 \times 2}{3 \times 3 \times 3}\right)^{\frac{1}{3}}$

$= \left(\dfrac{2^3}{3^3}\right)^{\frac{1}{3}}$

$= \dfrac{(2^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}}$

$= \dfrac{2}{3}$

(iii) $\sqrt{\dfrac{\mathbf{625x^3y^4}}{\mathbf{25xy^2}}}$

Solution:

$\sqrt{\dfrac{625x^3y^4}{25xy^2}}$

$= \sqrt{25x^2y^2}$

$= (25x^2y^2)^{\frac{1}{2}}$

$= (25)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}(y^2)^{\frac{1}{2}}$

$= 5xy$

(iv) $\sqrt{(\mathbf{3y-5})^2}$

Solution:

$\sqrt{(3y-5)^2}$

$= [(3y-5)^2]^{\frac{1}{2}}$

$= 3y - 5$

Ex # 2.3

(v) $\mathbf{6\sqrt{18}}$

Solution:

$6\sqrt{18}$

$= 6(18)^{\frac{1}{2}}$

$= 6(3 \times 3 \times 2)^{\frac{1}{2}}$

$= 6(3^2 \times 2)^{\frac{1}{2}}$

$= 6(3^2)^{\frac{1}{2}}(2)^{\frac{1}{2}}$

$= 6(3)\sqrt{2}$

$= 18\sqrt{2}$

(vi) $\sqrt[3]{\mathbf{54x^3y^3z^2}}$

Solution:

$\sqrt[3]{54x^3y^3z^2}$

$= (54x^3y^3z^2)^{\frac{1}{3}}$

$= (54)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^3)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$

$= (3 \times 3 \times 3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3^3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}}(2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$

$= (3)(x)(y)(2)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$

$= 3xy(2z^2)^{\frac{1}{3}}$

$= 3xy\sqrt[3]{2z^2}$

Ex # 2.4

Base

جس کے اوپر power ہو اسے Base کہتے ہیں۔

Exponent /Power

Base کے اوپر جو چھوٹا سا نمبر ہوتا ہے اسے power کہتے ہیں۔ اس کو index بھی کہتے ہیں۔

Co-efficient

Base کے Left طرف جو نمبر ہوتا ہے اسے Co-efficient کہتے ہیں۔

Base اور Co-efficient آپس میں Multiply ہوتے ہیں

$4x^2$ Base: x Power: 2 Co-efficient: 4	$5y^{-3}$ Base: y Power: -3 Co-efficient: 5	$-2y^3$ Base: y Power: 3 Co-efficient: -2
x Base: x Power: 1 Co-efficient: 1	x^3 Base: x Power: 3 Co-efficient: 1	$5z$ Base: z Power: 1 Co-efficient: 5

Note:

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\qquad \frac{1}{3^{-3}} = 3^3 = 27$

$-4x^{-2} = \frac{-4}{x^2}$ $\qquad (a+b)^{-1} = \frac{1}{(a+b)}$

Laws of Exponents

Multiplication of Same Bases

To multiply powers of the same base, keep the same base and add the exponents.

اگر ایک جیسے bases آپس میں multiply ہوتے ہیں تو:

- ❖ **Co-efficient کو multiply کریں گے**
- ❖ **Base ایک لکھیں گے**
- ❖ **Powers کو Add کریں گے**

Example:

$$a^m . a^n = a^{m+n}$$

Ex # 2.4

Multiplication of Different Bases

When different bases are multiplied just multiply the co-efficient or constant.

اگر مختلف bases آپس میں multiply ہوتے ہیں تو صرف Co-efficient کو multiply کریں گے

Law of Quotient

To divide two expressions with the same bases and different exponents, keep the same base and subtract the exponents.

جب fraction میں ایک جیسے bases ہوں تو اس base کو اوپر لے جائیں گے لیکن اس کے power کا sign تبدیل ہو جائے گا۔

- ❖ اگر plus ہو گا تو minus ہو جائے گا
- ❖ اگر minus ہو گا تو plus ہو جائے گا

Law of Power of Power

To raise an exponential expression to a power, keep the same base multiply the exponents.

جب کسی بریکٹ کے اوپر Power آجائیں تو اس کو تمام Bases کے Powers کے ساتھ Multiply کریں گے۔

اگر Base یا Co-efficient کے ساتھ minus کا sign ہو تو:

1) جب power میں even نمبر ہو تو expression کے ساتھ plus کا sign لگائیں گے۔

$(-x)^{22} = x^{22}$ $\qquad (-4y)^2 = 16y^2$

2) جب power میں Odd نمبر ہو تو expression کے ساتھ plus کا minus لگائیں گے۔

$(-x)^{25} = -x^{25}$ $\qquad (-2y)^3 = -8y^3$

Zero Exponent Rule

Any non-zero number raised to the zero power equals one.

کسی بھی Base کا Power اگر Zero ہو تو 1 کے برابر ہو گا۔

$100^0 = 1$ and $(xy)^0 = 1$

Ex # 2.4

Page # 67

Q1: **Write the base, exponent and value of the following.**

(i) $(2)^{-9} = \frac{1}{1024}$

$base = 2,\ Exponent = -9, value = \frac{1}{1024}$

(ii) $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

$base = \frac{a}{b},\ Exponent = p,\ \ value = \frac{a^p}{b^p}$

(iii) $(-4)^2 = 16$

$base = -4,\ \ Exponent = 2,\ \ value = 16$

Q2: 67 **If a, b denote the real numbers then simplify the following.**

(i) $a^3 \times a^5$

Solution:

$a^3 \times a^5$

$= a^{3+5}$

$= a^8$

(ii) $\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{-\frac{2}{3}}$

Solution:

$\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{-\frac{2}{3}}$

$= \left(\frac{b}{a}\right)^{\frac{3}{2}-\frac{2}{3}}$

$= \left(\frac{b}{a}\right)^{\frac{9-4}{6}}$

$= \left(\frac{b}{a}\right)^{\frac{5}{6}}$

(iii) $(-a)^4 \times (-a)^3$

Solution:

$(-a)^4 \times (-a)^3$

$= (-a)^{4+3}$

$= (-a)^7$

$= -a^7$

(iv) $(-2a^2b^3)^3$

Solution:

$(-2a^2b^3)^3$

$= (-2)^3a^{2\times3}b^{3\times3}$

$= -8a^6b^9$

(v) $a^3(-2b)^2$

Solution:

$= a^3(-2b)^2$

$= a^3(-2)^2(b)^2$

$= a^3 \times 4b^2$

$= 4a^3b^2$

(vi) $(a^2b)(a^2b)$

Solution:

$(a^2b)(a^2b)$

$= a^{2+2}b^{1+1}$

$= a^4b^2$

(vii) $\frac{a^0.b^0}{2}$

Solution:

$\frac{a^0.b^0}{2}$

$= \frac{1\times1}{2}$

$= \frac{1}{2}$

(viii) $(-3a^2b^2)^2$

Solution:

$(-3a^2b^2)^2$

$= (-3)^2a^{2\times2}b^{2\times2}$

$= 9a^4b^4$

Ex # 2.4

(ix) $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

Solution:

$\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

$= \frac{a^{2\times\frac{3}{2}}}{b^{4\times\frac{3}{2}}}$

$= \frac{a^{1\times3}}{b^{2\times3}}$

$= \frac{a^3}{b^6}$

Q3: Simplify the following.

(i) $\frac{7^6}{7^4}$

Solution:

$\frac{7^6}{7^4}$

$= 7^6.7^{-4}$

$= 7^{6-4}$

$= 7^2$

(ii) $\frac{2^4.5^3}{10^2}$

Solution:

$\frac{2^4.5^3}{10^2}$

$= \frac{2^4.5^3}{(2\times5)^2}$

$= \frac{2^4.5^3}{2^2.5^2}$

$= 2^4.5^3.2^{-2}.5^{-2}$

$= 2^{4-2}.5^{3-2}$

$= 2^2.5^1$

$= 4\times5$

$= 20$

Ex # 2.4

(iii) $\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

Solution:

$\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

$= \frac{(a+b)^{2\times3}.(c+d)^{3\times3}}{(a+b)^{1\times3}.(c+d)^{2\times3}}$

$= \frac{(a+b)^6.(c+d)^9}{(a+b)^3.(c+d)^6}$

$= (a+b)^6.(c+d)^9.(a+b)^{-3}.(c+d)^{-6}$

$= (a+b)^{6-3}.(c+d)^{9-6}$

$= (a+b)^3.(c+d)^3$

(iv) $(\sqrt[3]{a})^{\frac{1}{2}}$

Solution:

$(\sqrt[3]{a})^{\frac{1}{2}}$

$= \left(a^{\frac{1}{3}}\right)^{\frac{1}{2}}$

$= a^{\frac{1}{3}\times\frac{1}{2}}$

$= a^{\frac{1}{6}}$

(v) $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

Solution:

$\sqrt[5]{x^5}.\sqrt[4]{x^4}$

$= (x^5)^{\frac{1}{5}}(x^4)^{\frac{1}{4}}$

$= (x)^{5\times\frac{1}{5}}.(x)^{4\times\frac{1}{4}}$

$= x.x$

$= x^2$

Ex # 2.4

Q4: 67 **Simplify the following in such a way that no answers should contain fractional or negative exponent.**

(i) $\left(\frac{25}{81}\right)^{\frac{1}{2}}$

Solution:

$$\left(\frac{25}{81}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5\times 5}{9\times 9}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5^2}{9^2}\right)^{\frac{1}{2}}$$

$$= \frac{5^{2\times\frac{1}{2}}}{9^{2\times\frac{1}{2}}}$$

$$= \frac{5}{9}$$

(ii) $\dfrac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$

Solution:

$$\frac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$$

$$= \frac{(ab)^{\frac{1}{b}}}{((ab)^{-1})^{\frac{1}{a}}}$$

$$= \frac{(ab)^{\frac{1}{b}}}{(ab)^{-\frac{1}{a}}}$$

$$= (ab)^{\frac{1}{b}}.(ab)^{\frac{1}{a}}$$

$$= (ab)^{\frac{1}{b}+\frac{1}{a}}$$

$$= (ab)^{\frac{a+b}{ba}}$$

$$= (ab)^{\frac{a+b}{ab}}$$

$$= a^{\frac{a+b}{ab}}.b^{\frac{a+b}{ab}}$$

Ex # 2.4

(iii) $\dfrac{2^{p+1}.3^{2p-q}.5^{p+q}.6^{q}}{6^{p}.10^{q+2}.15^{p}}$

Solution:

$$\frac{2^{p+1}.3^{2p-q}.5^{p+q}.6^{q}}{6^{p}.10^{q+2}.15^{p}}$$

$$= \frac{2^{p+1}.3^{2p-q}.5^{p+q}.(2\times 3)^{q}}{(2\times 3)^{p}.(2\times 5)^{q+2}.(3\times 5)^{p}}$$

$$= \frac{2^{p+1}.3^{2p-q}.5^{p+q}.2^{q}.3^{q}}{2^{p}.3^{p}.2^{q+2}.5^{q+2}.3^{p}.5^{p}}$$

$$= \frac{2^{p+1+q}.3^{2p-q+q}.5^{p+q}}{2^{p+q+2}.3^{p+p}.5^{q+2+p}}$$

$$= \frac{2^{p+1+q}.3^{2p}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{q+2+p}}$$

$$= 2^{p+1+q}.3^{2p}.5^{p+q}.2^{-p-q-2}.3^{-2p}.5^{-q-2-p}$$

$$= 2^{p+1+q-p-q-2}.3^{2p-2p}.5^{p+q-q-2-p}$$

$$= 2^{1-2}.3^{0}.5^{-2}$$

$$= 2^{-1}.3^{0}.5^{-2}$$

$$= \frac{1}{2}\times 1\times\frac{1}{5^2}$$

$$= \frac{1}{2}\times 1\times\frac{1}{25}$$

$$= \frac{1}{50}$$

(iv) $\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$

Solution:

$$\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$$

$$= (x^p.x^{-q})^{p+q}(x^q.x^{-r})^{q+r}(x^r.x^{-p})^{r+p}$$

$$= (x^{p-q})^{p+q}(x^{q-r})^{q+r}(x^{r-p})^{r+p}$$

$$= (x)^{(p-q)(p+q)}.(x)^{(q-r)(q+r)}.(x)^{(r-p)(r+p)}$$

$$= (x)^{p^2-q^2}.(x)^{q^2-r^2}.(x)^{r^2-p^2}$$

$$= x^{p^2-q^2+q^2-r^2+r^2-p^2}$$

$$= x^0$$

$$= 1$$

Ex # 2.4

Q5: 67 **Prove that** $\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$

Solution:

$$\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$$

L.H.S

$$= \left(\frac{(2^2)^5.(2^6)^3.2^3}{(2^3)^5.(2^7)^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10}.2^{18}.2^3}{2^{15}.2^{14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10+18+3}}{2^{15+14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{31}}{2^{29}}\right)^{\frac{1}{2}}$$

$$= (2^{31-29})^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}}$$

$$= 2^{2\times\frac{1}{2}}$$

$$= 2$$

=R.H.S

Ex # 2.5

Complex Number

A number of the form $a + bi$ where a and b are real numbers is called complex number where "a" is called real part and "b" is called imaginary part.

Conjugate of a Complex Numbers

A conjugate of a complex number is obtained by changing the sign of imaginary part. The conjugate of $a + bi$ is $a - bi$ or the conjugate of $a + bi$ is denoted by $\overline{a + bi} = a - bi$.

Ex # 2.5

Equality of Two Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then $Z_1 = Z_2$ if real parts are equal i.e. $a = c$ and imaginary parts are equal i.e. $b = d$.

Operation on Complex Numbers

Addition of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1 + Z_2 = (a + bi) + (c + di)$

$Z_1 + Z_2 = a + bi + c + di$

$Z_1 + Z_2 = a + c + bi + di$

$Z_1 + Z_2 = (a + c) + (b + d)i$

Subtraction of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1 - Z_2 = (a + bi) - (c + di)$

$Z_1 - Z_2 = a + bi - c - di$

$Z_1 - Z_2 = a - c + bi - di$

$Z_1 - Z_2 = (a - c) + (b - d)i$

Multiplication of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1.Z_2 = (a + bi)(c + di)$

$Z_1.Z_2 = ac + adi + bci + bdi^2$

$Z_1.Z_2 = ac + (ad + bc)i + bd(-1)$ as $i^2 = -1$

$Z_1.Z_2 = ac + (ad + bc)i - bd$

$Z_1.Z_2 = (ac - bd) + (ad + bc)i$

Division of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di}$$

Multiply and Divide by $c - di$

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di}$$

$$\frac{Z_1}{Z_2} = \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

$$\frac{Z_1}{Z_2} = \frac{ac - adi + bci - bdi^2}{c^2 - (di)^2}$$

Ex # 2.5

$$\frac{Z_1}{Z_2}=\frac{ac+bci-adi-bd(-1)}{c^2-d^2i^2} \quad As\ i^2=-1$$
$$\frac{Z_1}{Z_2}=\frac{ac+(bc-ad)i+bd}{c^2-d^2(-1)}$$
$$\frac{Z_1}{Z_2}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$
$$\frac{Z_1}{Z_2}=\frac{(ac+bd)}{c^2+d^2}+\frac{(bc-ad)i}{c^2+d^2}$$

Ex # 2.5

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Q1: Add the following complex number

(i) $8+9i, 5+2i$

Solution:

$8+9i, 5+2i$

Let $Z_1=8+9i$

And $Z_2=5+2i$

Now

$Z_1+Z_2=(8+9i)+(5+2i)$
$Z_1+Z_2=8+9i+5+2i$
$Z_1+Z_2=8+5+9i+2i$
$Z_1+Z_2=13+11i$

(ii) $6+3i, 3-5i$

Solution:

$6+3i, 3-5i$

Let $Z_1=6+3i$

And $Z_2=3-5i$

Now

$Z_1+Z_2=(6+3i)+(3-5i)$
$Z_1+Z_2=6+3i+3-5i$
$Z_1+Z_2=6+3+3i-5i$
$Z_1+Z_2=9-2i$

(iii) $2i+3, 8-5\sqrt{-1}$

Solution:

$2i+3, 8-5\sqrt{-1}$

Let $Z_1=2i+3$

And $Z_2=8-5\sqrt{-1}$

$8-5i \quad \therefore \sqrt{-1}=i$

Ex # 2.5

Now

$Z_1+Z_2=(2i+3)+(8-5i)$
$Z_1+Z_2=2i+3+8-5i$
$Z_1+Z_2=3+8+2i-5i$
$Z_1+Z_2=11-3i$

(iv) $\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Solution:

$\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Let $Z_1=\sqrt{3}+\sqrt{2}i$

And $Z_2=3\sqrt{3}-2\sqrt{2}i$

Now

$Z_1+Z_2=(\sqrt{3}+\sqrt{2}i)+(3\sqrt{3}-2\sqrt{2}i)$
$Z_1+Z_2=\sqrt{3}+\sqrt{2}i+3\sqrt{3}-2\sqrt{2}i$
$Z_1+Z_2=\sqrt{3}+3\sqrt{3}+\sqrt{2}i-2\sqrt{2}i$
$Z_1+Z_2=4\sqrt{3}-\sqrt{2}i$

Q2: Subtract:

(i) $-2+3i$ from $6-3i$

Solution:

$-2+3i$ from $6-3i$

Let $Z_1=-2+3i$

And $Z_2=6-3i$

Now

$Z_2-Z_1=(6-3i)-(-2+3i)$
$Z_2-Z_1=6-3i+2-3i$
$Z_2-Z_1=6+2-3i-3i$
$Z_2-Z_1=8-6i$

(ii) $9+4i$ from $9-8i$

Solution:

$9+4i$ from $9-8i$

Let $Z_1=9+4i$

And $Z_2=9-8i$

Now

$Z_2-Z_1=(9-8i)-(9+4i)$
$Z_2-Z_1=9-8i-9-4i$
$Z_2-Z_1=9-9-8i-4i$
$Z_2-Z_1=0-12i$
$Z_2-Z_1=-12i$

Ex # 2.5

(iii) $\mathbf{1 - 3i}$ **from** $\mathbf{8 - i}$

Solution:

$1 - 3i$ from $8 - i$

Let $Z_1 = 1 - 3i$

And $Z_2 = 8 - i$

Now

$Z_2 - Z_1 = (8 - i) - (1 - 3i)$

$Z_2 - Z_1 = 8 - i - 1 + 3i$

$Z_2 - Z_1 = 8 - 1 - i + 3i$

$Z_2 - Z_1 = 7 + 2i$

(iv) $6 - 7i$ from $6 + 7i$

Solution:

$6 - 7i$ from $6 + 7i$

Let $Z_1 = 6 - 7i$

And $Z_2 = 6 + 7i$

Now

$Z_2 - Z_1 = (6 + 7i) - (6 - 7i)$

$Z_2 - Z_1 = 6 + 7i - 6 + 7i$

$Z_2 - Z_1 = 6 - 6 + 7i + 7i$

$Z_2 - Z_1 = 0 + 14i$

$Z_2 - Z_1 = 14i$

Q3: Multiply the following complex numbers

(i) $\mathbf{1 + 2i, 3 - 8i}$

Solution:

$1 + 2i, 3 - 8i$

Let $Z_1 = 1 + 2i$

And $Z_2 = 3 - 8i$

Now

$Z_1.Z_2 = (1 + 2i)(3 - 8i)$

$Z_1.Z_2 = 1(3 - 8i) + 2i(3 - 8i)$

$Z_1.Z_2 = 3 - 8i + 6i - 16i^2$

$Z_1.Z_2 = 3 - 2i - 16(-1)$

$Z_1.Z_2 = 3 - 2i + 16$

$Z_1.Z_2 = 3 + 16 - 2i$

$Z_1.Z_2 = 19 - 2i$

(ii) $\mathbf{2i, 4 - 7i}$

Solution:

$2i, 4 - 7i$

Let $Z_1 = 2i$

And $Z_2 = 4 - 7i$

Ex # 2.5

Now

$Z_1.Z_2 = (2i)(4 - 7i)$

$Z_1.Z_2 = 2i(4 - 7i)$

$Z_1.Z_2 = 8i - 14i^2$

$Z_1.Z_2 = 8i - 14(-1)$

$Z_1.Z_2 = 8i + 14$

$Z_1.Z_2 = 14 + 8i$

(iii) $\mathbf{5 - 3i, 2 - 4i}$

Solution:

$5 - 3i, 2 - 4i$

Let $Z_1 = 5 - 3i$

And $Z_2 = 2 - 4i$

Now

$Z_1.Z_2 = (5 - 3i)(2 - 4i)$

$Z_1.Z_2 = 5(2 - 4i) - 3i(2 - 4i)$

$Z_1.Z_2 = 10 - 20i - 6i + 12i^2$

$Z_1.Z_2 = 10 - 26i + 12(-1)$

$Z_1.Z_2 = 10 - 26i - 12$

$Z_1.Z_2 = 10 - 12 - 26i$

$Z_1.Z_2 = -2 - 26i$

(iv) $\mathbf{\sqrt{2} + i, 1 - \sqrt{2}i}$

Solution:

$\sqrt{2} + i, 1 - \sqrt{2}i$

Let $Z_1 = \sqrt{2} + i$

And $Z_2 = 1 - \sqrt{2}i$

Now

$Z_1.Z_2 = (\sqrt{2} + i)(1 - \sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}(1 - \sqrt{2}i) + i(1 - \sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2} - \sqrt{2 \times 2}i + 1i - \sqrt{2}i^2$

$Z_1.Z_2 = \sqrt{2} - 2i + 1i - \sqrt{2}(-1)$

$Z_1.Z_2 = \sqrt{2} - i + \sqrt{2}$

$Z_1.Z_2 = \sqrt{2} + \sqrt{2} - i$

$Z_1.Z_2 = 2\sqrt{2} - i$

Ex # 2.5

Q4: Divide the first complex number by the second.

(i) $\boldsymbol{Z_1 = 2 + i, Z_2 = 5 - i}$

Solution:

$Z_1 = 2 + i, Z_2 = 5 - i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i}$$

Multiply and divide by $5 + i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i} \times \frac{5+i}{5+i}$$

$$\frac{Z_1}{Z_2} = \frac{(2+i)(5+i)}{(5-i)(5+i)}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 2i + 5i + i^2}{(5)^2 - (i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 7i + (-1)}{25 - i^2}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 7i - 1}{25 - (-1)}$$

$$\frac{Z_1}{Z_2} = \frac{10 - 1 + 7i}{25 + 1}$$

$$\frac{Z_1}{Z_2} = \frac{9 + 7i}{26}$$

$$\frac{Z_1}{Z_2} = \frac{9}{26} + \frac{7}{26}i$$

(ii) $\boldsymbol{Z_1 = 3i + 4, Z_2 = 1 - i}$

Solution:

$Z_1 = 3i + 4$

$4 + 3i$

$Z_2 = 1 - i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i}$$

Multiply and divide by $1 + i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i} \times \frac{1+i}{1+i}$$

Ex # 2.5

$$\frac{Z_1}{Z_2} = \frac{(4+3i)(1+i)}{(1-i)(1+i)}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 4i + 3i + 3i^2}{(1)^2 - (i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 7i + 3(-1)}{1 - i^2}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 7i - 3}{1 - (-1)}$$

$$\frac{Z_1}{Z_2} = \frac{4 - 3 + 7i}{1 + 1}$$

$$\frac{Z_1}{Z_2} = \frac{1 + 7i}{2}$$

$$\frac{Z_1}{Z_2} = \frac{1}{2} + \frac{7}{2}i$$

Q5: Perform the indicated operations and reduce to the form $\boldsymbol{a + bi}$

(i) $\boldsymbol{(4 - 3i) + (2 - 3i)}$

Solution:

$(4 - 3i) + (2 - 3i)$
$= 4 - 3i + 2 - 3i$
$= 4 + 2 - 3i - 3i$
$= 6 - 6i$

(ii) $\boldsymbol{(5 - 2i) - (4 - 7i)}$

Solution:

$(5 - 2i) - (4 - 7i)$
$= 5 - 2i - 4 + 7i$
$= 5 - 4 - 2i + 7i$
$= 1 + 5i$

(iii) $\boldsymbol{2i(4 - 5i)}$

Solution:

$2i(4 - 5i)$
$= 2i - 10i^2$
$= 2i - 10(-1)$
$= 2i + 10$
$= 10 + 2i$

Ex # 2.5

(iv) $(\mathbf{2-3i}) \div (\mathbf{4-5i})$

Solution:

$(2-3i) \div (4-5i)$

$=\dfrac{2-3i}{4-5i}$

Multiply and divide by $4+5i$

$=\dfrac{2-3i}{4-5i}\times\dfrac{4+5i}{4+5i}$

$=\dfrac{(2-3i)(4+5i)}{(4-5i)(4+5i)}$

$=\dfrac{8+10i-12i-15i^2}{(4)^2-(5i)^2}$

$=\dfrac{8-2i-15(-1)}{16-25i^2}$

$=\dfrac{8-2i+15}{16-25(-1)}$

$=\dfrac{8+15-2i}{16+25}$

$=\dfrac{23-2i}{41}$

$=\dfrac{23}{41}-\dfrac{2}{41}i$

Q6: Find the complex conjugate of the following complex numbers.

(i) $-8-3i$

The complex conjugate of $-8-3i$ is $-8+3i$

(i) $\mathbf{-4+9i}$

The complex conjugate of $-4+9i$ is $-4-9i$

(iii) $\mathbf{7+6i}$

The complex conjugate of $7+6i$ is $7-6i$

(iv) $\mathbf{\sqrt{5}-i}$

The complex conjugate of $\sqrt{5}-i$ is $\sqrt{5}+i$

Review Ex # 2

Page # 73

Q3: Simplify each of the following.

(i) $\left(\dfrac{-2}{3}\right)^3$

Solution:

$\left(\dfrac{-2}{3}\right)^3$

$=\dfrac{(-2)^3}{(3)^3}$

$=\dfrac{-8}{27}$

(ii) $(-2)^3.(3)^2$

Solution:

$(-2)^3.(3)^2$

$=-8\times 9$

-72

(iii) $-3\sqrt{48}$

Solution:

$-3\sqrt{48}$

$-3\sqrt{4\times 4\times 3}$

$-3\sqrt{4\times 4}\times\sqrt{3}$

$-3\times 4\sqrt{3}$

$-12\sqrt{3}$

(iv) $\dfrac{5}{\sqrt[3]{9}}$

Solution:

$\dfrac{5}{\sqrt[3]{9}}$

$=\dfrac{5}{(9)^{\frac{1}{3}}}$

$=\dfrac{5}{(3^2)^{\frac{1}{3}}}$

$=\dfrac{5}{(3)^{\frac{2}{3}}}$

Review Ex # 2

Multiply and Divide by $\sqrt[3]{3}$

$$\frac{5}{(3)^{\frac{2}{3}}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$

$$\frac{5 \times \sqrt[3]{3}}{(3)^{\frac{2}{3}} \times (3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{2}{3}+\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{3}$$

Q4: **Multiply $8i, \ -8i$**

Solution:

$8i, \ -8i$

Now

$(8i)(-8i) = -64i^2$

$= -64(-1)$

$= 64$

Q5: **Divide $2 - 5i$ by $1 - 6i$**

Solution:

$$\frac{2-5i}{1-6i}$$

i

Multiply and divide by $1 + 6i$

$$= \frac{2-5i}{1-6} \times \frac{1+6i}{1+6i}$$

$$= \frac{(2-5i)(1+6i)}{(1-6i)(1+6i)}$$

$$= \frac{2+12i-5i-30i^2}{(1)^2-(6i)^2}$$

$$= \frac{2+7i-30(-1)}{1-36i^2}$$

$$= \frac{2+7i+30}{1-36(-1)}$$

Review Ex # 2

$$= \frac{2+30+7i}{1+36}$$

$$= \frac{32+7i}{37}$$

$$= \frac{32}{37} - \frac{7}{37}i$$

Q7: **Use laws of exponents to simplify:**

$$\frac{(\mathbf{81})^{n}.\mathbf{3}^{5} + (\mathbf{3})^{4n-1}(\mathbf{243})}{(\mathbf{9}^{2n})(\mathbf{3}^{3})}$$

Solution:

$$\frac{(81)^n.3^5 + (3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

$$= \frac{(3^4)^n.3^5 + 3^{4n-1}.(3^5)}{(3^2)^{2n}(3^3)}$$

$$= \frac{3^{4n}.3^5 + 3^{4n}.3^{-1}.3^5}{3^{4n}.3^3}$$

$$= \frac{3^{4n}.3^5(1+3^{-1})}{3^{4n}.3^3}$$

$$= \frac{3^{4n}.3^3.3^2(1+3^{-1})}{3^{4n}.3^3}$$

$= 3^2(1+3^{-1})$

$$= 9\left(1+\frac{1}{3}\right)$$

$$= 9\left(\frac{3+1}{3}\right)$$

$$= 9\left(\frac{4}{3}\right)$$

$= 3 \times 4$

$= 12$

Q6: **Name the property used**

$$\mathbf{7} \times \frac{\mathbf{1}}{\mathbf{7}} = \frac{\mathbf{1}}{\mathbf{7}} \times \mathbf{7} = \mathbf{1}$$

Answer:

Multiplicative Property

UNIT # 3
LOGARITHM

Exercise # 3.1

SCIENTIFIC NOTATION:

Scientific notation is a way of writing numbers that are too big or too small to be easily written in decimal form.

Representation

The positive number "x" is represented in scientific notation as the product of two numbers where the first number "a" is a real number greater than 1 and less than 10 and the second is the integral power of "n" of 10.

$$x = a \times 10^n$$

Rules for Standard Notation to Scientific Notation

(i) In a given number, place the decimal after first non-zero digit.

(ii) If the decimal point is moved towards left, then the power of "10" will be positive.

(iii) If the decimal is moved towards right, then the power of "10" will be negative.
The numbers of digits through which the decimal point has been moved will be the exponent.

Rules for Standard Notation to Scientific Notation

(i) If the exponent of 10 is Positive, move the decimal towards Right.

(ii) If the exponent of 10 is Negative, move the decimal toward Left.

(iii) Move the decimal point to the same number of digits as the exponent of 10.

Example # 7 Page # 80

How many miles does light travel in 1 day? The speed of the light is 186,000 mi/ sec. write the answer in scientific notation.

Solution:

$$Time = t = 1\ day = 24\ hr$$
$$t = 24 \times 60 \times 60\ sec = 86400$$
$$t = 8.64 \times 10^4\ sec$$
$$Speed = v = 186000\ mi/sec$$
$$v = 1.86 \times 10^5\ mi/sec$$

As we know that

$$s = vt$$

Put the values

$$s = 1.86 \times 10^5 \times 8.64 \times 10^4$$
$$s = 1.86 \times 8.64 \times 10^5 \times 10^4$$
$$s = 16.0704 \times 10^{5+4}$$
$$s = 16.0704 \times 10^9$$
$$s = 1.60704 \times 10^1 \times 10^9$$
$$s = 1.60704 \times 10^{10}$$

Thus light travels $1.60704 \times 10^1 \times 10^9$ miles in a day

Exercise # 3.1
Page # 80

Q1: Write each number in scientific notation.

(i) 405,000

Solution:

405,000

In Scientific Form:

$$4.05 \times 10^4$$

(ii) 1,670,000

Solution:

1,670,000

In Scientific Form:

$$1.67 \times 10^6$$

(iii) 0.00000039

Solution:

0.00000039

In Scientific Form:

$$3.9 \times 10^{-7}$$

(iv) 0.00092

Solution:

0.00092

In Scientific Form:

$$9.2 \times 10^{-4}$$

Ex # 3.1

(v) **234,600,000,000**

Solution:

234,600,000,000

In Scientific Form:

$$2.346 \times 10^{11}$$

(vi) **8,904,000,000**

Solution:

8,904,000,000

In Scientific Form:

$$8.904 \times 10^{9}$$

(vii) **0.00104**

Solution:

0.00104

In Scientific Form:

1.04×10^{-3}

(viii) **0.00000000514**

Solution:

0.00000000514

In Scientific Form:

$$5.14 \times 10^{-9}$$

(ix) $\mathbf{0.05 \times 10^{-3}}$

Solution:

$$0.05 \times 10^{-3}$$

In Scientific Form:

$$5.0 \times 10^{-2} \times 10^{-3}$$
$$5.0 \times 10^{-2-3}$$
$$5.0 \times 10^{-5}$$

Q2: Write each number in standard notation.

(i) $\mathbf{8.3 \times 10^{-5}}$

Solution:

8.3×10^{-5}

In Standard Form:

0.000083

(ii) $\mathbf{4.1 \times 10^{6}}$

Solution:

4.1×10^{6}

In Standard Form:

410000

Ex # 3.1

(iii) $\mathbf{2.07 \times 10^{7}}$

Solution:

2.07×10^{7}

In Standard Form:

20700000

(iv) $\mathbf{3.15 \times 10^{-6}}$

Solution:

3.15×10^{-6}

In Standard Form:

0.00000315

(v) $\mathbf{6.27 \times 10^{-10}}$

Solution:

6.27×10^{-10}

In Standard Form:

0.000000000627

(vi) $\mathbf{5.41 \times 10^{-8}}$

Solution:

5.41×10^{-8}

In Standard Form:

0.0000000541

(vii) $\mathbf{7.632 \times 10^{-4}}$

Solution:

7.632×10^{-4}

In Standard Form:

0.0007632

(viii) $\mathbf{9.4 \times 10^{5}}$

Solution:

9.4×10^{5}

In Standard Form:

940000

(ix) $\mathbf{-2.6 \times 10^{9}}$

Solution:

-2.6×10^{9}

In Standard Form:

-2600000000

Ex # 3.1

Q3: How long does it take light to travel to Earth from the sun? The sun is 9.3×10^7 miles from Earth, and light travels 1.86×10^5 mi/s.

Solution:

Given:

Distance between earth and sun $= 9.3 \times 10^7$ miles

Speed of light $= 1.86 \times 10^5$ mi/s

As we have:

$s = vt$

$\frac{s}{v} = t$

Or

$t = \frac{s}{v}$

Put the values:

$t = \frac{9.3 \times 10^7}{1.86 \times 10^5}$

$t = 5 \times 10^7 \times 10^{-5}$

$t = 5 \times 10^{7-5}$

$t = 5 \times 10^2$

$t = 500 \ sec$

$t = 480 \ \text{sec} + 20 \ sec$

$t = 8 \ \text{min} \ 20 \ sec$

Exercise # 3.2

Logarithm

If $a^x = y$ then the index x is called the logarithm of y to the base a and written as $\log_a y = x$.

We called $\log_a y = x$ like log of y to the base a equal to x.

Logarithm Form	Exponential Form
$\log_a y = x$	$a^x = y$
$\log_8 64 = 2$	$8^2 = 64$

Ex # 3.2

Page # 83

Q1: Write the following in logarithm form.

(i) $4^4 = 256$

Solution:

$4^4 = 256$

In logarithm form

$\log_4 256 = 4$

(ii) $2^{-6} = \frac{1}{64}$

Solution:

$2^{-6} = \frac{1}{64}$

In logarithm form

$\log_2 \frac{1}{64} = -6$

(iii) $10^0 = 1$

Solution:

$10^0 = 1$

In logarithm form

$\log_{10} 1 = 0$

(iv) $x^{\frac{3}{4}} = y$

Solution:

$x^{\frac{3}{4}} = y$

In logarithm form

$\log_x y = \frac{3}{4}$

(v) $3^{-4} = \frac{1}{81}$

Solution:

$3^{-4} = \frac{1}{81}$

In logarithm form

$\log_3 \frac{1}{81} = -4$

(vi) $64^{\frac{2}{3}} = 16$

Solution:

$64^{\frac{2}{3}} = 16$

In logarithm form

$\log_{64} 16 = \frac{2}{3}$

Ex # 3.2

Q2: Write the following in exponential form.

(i) $\log_a\left(\frac{1}{a^2}\right) = -1$

Solution:

$\log_a\left(\frac{1}{a^2}\right) = -1$

In exponential form

$a^{-1} = \frac{1}{a^2}$

(ii) $\log_2 \frac{1}{128} = -7$

Solution:

$\log_2 \frac{1}{128} = -7$

In exponential form

$2^{-7} = \frac{1}{128}$

(iii) $\log_b 3 = 64$

Solution:

$\log_b 3 = 64$

In exponential form

$b^{64} = 3$

(iv) $\log_a a = 1$

Solution:

$\log_a a = 1$

In exponential form

$a^1 = 1$

(v) $\log_a 1 = 0$

Solution:

$\log_a 1 = 0$

In exponential form

$a^0 = 1$

(vi) $\log_4 \frac{1}{8} = \frac{-3}{2}$

Solution:

$\log_4 \frac{1}{8} = \frac{-3}{2}$

In exponential form

$4^{\frac{-3}{2}} = \frac{1}{8}$

Ex # 3.2

Q3: Solve:

(i) $\log_{\sqrt{5}} 125 = x$

Solution:

$\log_{\sqrt{5}} 125 = x$

In exponential form

$(\sqrt{5})^x = 125$

$\left(5^{\frac{1}{2}}\right)^x = 5 \times 5 \times 5$

$5^{\frac{x}{2}} = 5^3$

Now

$\frac{x}{2} = 3$

Multiply B.S by 2

$2 \times \frac{x}{2} = 2 \times 3$

$x = 6$

(ii) $\log_4 x = -3$

Solution:

$\log_4 x = -3$

In exponential form

$4^{-3} = x$

Now

$\frac{1}{4^3} = x$

$\frac{1}{4 \times 4 \times 4} = x$

$\frac{1}{64} = x$

Or

$x = \frac{1}{64}$

(iii) $\log_{81} 9 = x$

Solution:

$\log_{81} 9 = x$

In exponential form

$81^x = 9$

$(9^2)^x = 9^1$

$9^{2x} = 9^1$

Now $2x = 1$

Divide B.S by 2

$\frac{2x}{2} = \frac{1}{2}$

$2x = \frac{1}{2}$

Ex # 3.2

(iv) $\log_3(5x+1)=2$

Solution:

$\log_3(5x+1)=2$

In exponential form

$3^2=5x+1$

$9=5x+1$

Subtract 1 form B.S

$9-1=5x+1-1$

$8=5x$

Divide B.S by 5

$\frac{8}{5}=\frac{5x}{5}$

$\frac{8}{5}=x$

$x=\frac{8}{5}$

(v) $\log_2 x=7$

Solution:

$\log_2 x=7$

In exponential form

$2^7=x$

Now

$2\times2\times2\times2\times2\times2\times2=x$

$128=x$

$x=128$

(vi) $\log_x 0.25=2$

Solution:

$\log_x 0.25=2$

In exponential form

$x^2=0.25$

$x^2=\frac{25}{100}$

Taking square root on B.S

$\sqrt{x^2}=\sqrt{\frac{25}{100}}$

$x=\frac{5}{10}$

$x=\frac{1}{2}$

Ex # 3.2

(vii) $\log_x(0.001)=-3$

Solution:

$\log_x(0.001)=-3$

In exponential form

$x^{-3}=0.001$

$x^{-3}=\frac{1}{1000}$

$x^{-3}=\frac{1}{10^3}$

$x^{-3}=10^{-3}$

So

$x=10$

(viii) $\log_x\frac{1}{64}=-2$

Solution:

$\log_x\frac{1}{64}=-2$

In exponential form

$x^{-2}=\frac{1}{64}$

$x^{-2}=\frac{1}{8\times8}$

$x^{-2}=\frac{1}{8^2}$

$x^{-2}=8^{-2}$

So

$x=8$

(ix) $\log_{\sqrt{3}} x=16$

Solution:

$\log_{\sqrt{3}} x=16$

In exponential form

$(\sqrt{3})^{16}=x$

$\left(3^{\frac{1}{2}}\right)^{16}=x$

$3^{\frac{16}{2}}=x$

$3^8=x$

$3\times3\times3\times3\times3\times3\times3\times3=x$

$6561=x$

$x=6561$

Exercise # 3.3

COMMON LOGARITHM

Introduction

The common logarithm was invented by a British Mathematician Prof. Henry Briggs (1560-1631).

Definition

Logarithms having base 10 are called common logarithms or Briggs logarithms.

Note:

The base of logarithm is not written because it is considered to be 10.

Logarithm of the number consists of two parts. Characteristics and Mantissa

Example: 1.5377

Characteristics

The digit before the decimal point or Integral part is called characteristics

Mantissa

The decimal fraction part is mantissa.

In above example

1 is Characteristics and $.5377$ is Mantissa.

USE OF LOG TABLE TO FIND MANTISSA:

A logarithm table is divided into three parts.

(i) The first part of the table is the extreme left column contains number from10 to 99.

(ii) The second part of the table consists of 10 columns headed by $0, 1, 2, \ldots .9$. The number under these columns are taken to find mantissa.

(iii) The third part consists of small columns known as mean difference headed by $1, 2, 3, \ldots 9$. These columns are added to the Mantissa found in second column.

To Find Mantissa

Let we have an example: 763.5

Solution:

(i) First ignore the decimal point.

(ii) Take first two digits e.g. 76 and proceed along this row until we come to column headed by third digit 3 of the number which is 8825.

(iii) Now take fourth digit i.e. 5 and proceed along this row in mean difference column which is 5.

(iv) Now add $8825 + 3 = 8828$

Ex # 3.3

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Q1: **Find the characteristics of the common logarithm of each of the following numbers.**

(i) **57**

In Scientific form:

5.7×10^1

Thus Characteristics $= 1$

(ii) **7.4**

In Scientific form:

7.4×10^0

Thus Characteristics $= 0$

(iii) **5.63**

In Scientific form:

5.63×10^0

Thus Characteristics $= 0$

(iv) **56.3**

In Scientific form:

5.63×10^1

Thus Characteristics $= 1$

(v) **982.5**

In Scientific form:

9.825×10^2

Thus Characteristics $= 2$

(vi) **7824**

In Scientific form:

7.824×10^3

Thus Characteristics $= 3$

(vii) **186000**

In Scientific form:

1.86×10^5

Thus Characteristics $= 5$

viii. **0.71**

In Scientific form:

7.1×10^{-1}

Thus Characteristics $= -1$

Ex # 3.3

Q2: Find the following.

(i) $\log 87.2$

Solution:

$\log 87.2$

In Scientific form:

8.72×10^1

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9405$

Hence $\log 87.2 = 1.9405$

(ii) $\log 37300$

Solution:

$\log 37300$

In Scientific form:

3.73×10^4

Thus Characteristics $= 4$

To find Mantissa, using Log Table:

So Mantissa $= .5717$

Hence $\log 37300 = 4.5717$

(iii) $\log 753$

Solution:

$\log 753$

In Scientific form:

7.53×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .8768$

Hence $\log 753 = 2.8768$

(iv) $\log 9.21$

Solution:

$\log 9.21$

In Scientific form:

9.21×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .9643$

Hence $\log 9.21 = 0.9643$

Ex # 3.3

(v) $\log 0.00159$

Solution:

$\log 0.00159$

In Scientific form:

1.59×10^{-3}

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .2014$

Hence $\log 0.00159 = \bar{3}.2014$

(vi) $\log 0.0256$

Solution:

$\log 0.0256$

In Scientific form:

2.56×10^{-2}

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .4082$

Hence $\log 0.0256 = \bar{2}.4082$

(vii) $\log 6.753$

Solution:

$\log 6.753$

In Scientific form:

6.753×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table

Mantissa $= .8295$

Hence $\log 6.753 = 0.8295$

$R.W$
$8293 + 2$ $= 8295$

Q3: Find logarithms of the following numbers.

(i) 2476

Solution:

2476

$Let\ x = 2476$

Taking log on B.S

$\log x = \log 2476$

In Scientific form:

2.476×10^3

Thus Characteristics $= 3$

To find Mantissa, using Log Table

So Mantissa $= .3927 + 11$

Mantissa $= .3938$

Hence $\log 2476 = 3.3938$

$R.W$
$3927 + 11$ $= 3938$

Ex # 3.3

(ii) 2.4

Solution:

2.4

$Let\ x = 2.4$

Taking log on B.S

$\log x = \log 2.4$

In Scientific form:

2.4×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .3802$

Hence $\log 2.4 = 0.3802$

(iii) 92.5

Solution:

92.5

$Let\ x = 92.5$

Taking log on B.S

$\log x = \log 92.5$

In Scientific form:

9.25×10^1

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9661$

Hence $\log 92.5 = 1.9661$

(iv) 482.7

Solution:

482.7

$Let\ x = 482.7$

Taking log on B.S

$\log x = \log 482.7$

In Scientific form:

4.827×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .6836$

Hence $\log 482.7 = 2.6836$

$R.W$
$6830 + 6$ $= 6836$

Ex # 3.3

(v) 0.783

Solution:

0.783

$Let\ x = 0.783$

Taking log on B.S

$\log x = \log 0.783$

In Scientific form:

7.83×10^{-1}

Thus Characteristics $= -1$

To find Mantissa, using Log Table:

So Mantissa $= .8938$

Hence $\log 0.783 = \bar{1}.8938$

(vi) 0.09566

Solution:

0.09566

$Let\ x = 0.09566$

Taking log on B.S

$\log x = \log 0.09566$

In Scientific form:

9.566×10^{-2}

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .9808$

Hence $\log 0.09566 = \bar{2}.9808$

$R.W$
$9805 + 3$ $= 9808$

(vii) 0.006753

Solution:

0.006753

$Let\ x = 0.006753$

Taking log on B.S

$\log x = \log 0.006753$

In Scientific form:

6.753×10^{-3}

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .8295$

Hence $\log 0.006735 = \bar{3}.8295$

$R.W$
$8293 + 2$ $= 8295$

Ex # 3.3

(viii) **700**

Solution:

700

$Let\ x = 700$

Taking log on B.S

$\log x = \log 700$

In Scientific form:

7.00×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .8451$

Hence $\log 700 = 2.8451$

Exercise # 3.4

ANTI-LOGARITHM

If $\log x = y$ then x is the anti-logarithm of y and written as $x = anti - \log y$

Explanation with Example:

2.3456

(i) Here the digit before decimal point is Characteristics i.e. 2

(ii) And Mantissa$= .3456$

To find anti-log, we see Mantissa in Anti-log Table

(i) Take first two digits i.e. $.34$ and proceed along this row until we come to column headed by third digit 5 of the number which is 2213.

(ii) Now take fourth digit i.e. 6 and proceed along this row which is 3.

(iii) Now add $2213 + 3 = 2216$

So to find anti-log, write it in Scientific form like

$anti - \log 2.3456 = 2.2216 \times 10^{char}$

$anti - \log 2.3456 = 2.216 \times 10^2$

$anti - \log 2.3456 = 221.6$

Ex # 3.4

Page # 88

Q1: Find anti-logarithm of the following numbers.

(i) **1.2508**

Solution:

1.2508

$Let\ \log x = 1.2508$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 1.2508$

$x = \text{Anti} - \log 1.2508$

Characteristics $= 1$

Mantissa $= .2508$

$R.W$
1778+3 = 1781

So

$x = 1.781 \times 10^1$

$x = 17.81$

(ii) **0.8401**

Solution:

0.8401

$Let\ \log x = 0.8401$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 0.8401$

$x = \text{Anti} - \log 0.8401$

Characteristics $= 0$

Mantissa $= .8401$

$R.W$
6918+2 = 6920

So

$x = 6.920 \times 10^0$

$x = 6.920$

(iii) **2.540**

Solution:

2.540

$Let\ \log x = 2.540$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 2.540$

$x = \text{Anti} - \log 2.540$

Characteristics $= 2$

Mantissa $= .540$

So

$x = 3.467 \times 10^2$

$x = 346.7$

Ex # 3.4

(iv) $\bar{2}.2508$

Solution:

$\bar{2}.2508$

Let $\log x = \bar{2}.2508$

Taking anti-log on B.S

$Anti-\log(\log x) = \text{Anti}-\log \bar{2}.2508$

$x = \text{Anti}-\log \bar{2}.2508$

Characteristics $= -2$

Mantissa $= .2508$

So

$x = 1.781 \times 10^{-2}$

$x = 0.01781$

$R.W$
1778+3 = 1781

(v) $\bar{1}.5463$

Solution:

$\bar{1}.5463$

Let $\log x = \bar{1}.5463$

Taking anti-log on B.S

$Anti-\log(\log x) = \text{Anti}-\log \bar{1}.5463$

$x = \text{Anti}-\log \bar{1}.5463$

Characteristics $= -1$

Mantissa $= .5463$

So

$x = 3.518 \times 10^{-1}$

$x = 0.3518$

$R.W$
3516+2 = 3518

(vi) **3.5526**

Solution:

3.5526

Let $\log x = 3.5526$

Taking anti-log on B.S

$Anti-\log(\log x) = \text{Anti}-\log 3.5526$

$x = \text{Anti}-\log 3.5526$

Characteristics $= 3$

Mantissa $= .5526$

So

$x = 3.570 \times 10^{3}$

$x = 3570$

$R.W$
3565+5 = 3570

Ex # 3.4

Q2: **Find the values of x from the following equations:**

(i) $\log x = \bar{1}.8401$

Solution:

$\log x = \bar{1}.8401$

Taking anti – log on B.S

$\text{Anti}-\log(\log x) = \text{Anti}-\log \bar{1}.8401$

$x = \text{Anti}-\log \bar{1}.8401$

Characteristics $= -1$

Mantissa $= .8401$

So

$x = 6.920 \times 10^{-1}$

$x = 0.6920$

$R.W$
$6918 + 2 = 6920$

(ii) $\log x = 2.1931$

Solution:

$\log x = 2.1931$

Taking anti – log on B.S

$\text{Anti}-\log(\log x) = \text{Anti}-\log 2.1931$

$x = \text{Anti}-\log 2.1931$

Characteristics $= 2$

Mantissa $= .1931$

So

$x = 1.560 \times 10^{2}$

$x = 156.0$

$R.W$
$1560 + 0 = 1560$

(iii) $\log x = 4.5911$

Solution:

$\log x = 4.5911$

Taking anti – log on B.S

$\text{Anti}-\log(\log x) = \text{Anti}-\log 4.5911$

$x = \text{Anti}-\log 4.5911$

Characteristics $= 4$

Mantissa $= .5911$

So

$x = 3.900 \times 10^{4}$

$x = 39000.0$

$R.W$
$3899 + 1 = 3900$

Ex # 3.4

(i) $\log x = \bar{3}.0253$

Solution:

$\log x = \bar{3}.0253$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{3}.0253$

$x = \text{Anti} - \log \bar{3}.0253$

Characteristics $= -3$

Mantissa $= .0253$

So

$x = 1.060 \times 10^{-3}$

$x = 0.001060$

$R.W$
$1059 + 1$ $= 1060$

(ii) $\log x = 1.8716$

Solution:

$\log x = 1.8716$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 1.8716$

$x = \text{Anti} - \log 1.8716$

Characteristics $= 1$

Mantissa $= .8716$

So

$x = 7.440 \times 10^{1}$

$x = 74.40$

$R.W$
$7430 + 10$ $= 7440$

(iii) $\log x = \bar{2}.8370$

Solution:

$\log x = \bar{2}.8370$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{2}.8370$

$x = \text{Anti} - \log \bar{2}.8370$

Characteristics $= -2$

Mantissa $= .8370$

So

$x = 6.871 \times 10^{-2}$

$x = 0.06781$

Ex # 3.5

LAWS OF LOGARITHM

(i) $\log_a mn = \log_a m + \log_a n$

or $\log mn = \log m + \log n$

Example:

$\log 2 \times 3 = \log 2 + \log 3$

(ii) $\log_a \dfrac{m}{n} = \log_a m - \log_a n$

or $\log \dfrac{m}{n} = \log m - \log n$

Example:

$\log \dfrac{3}{5} = \log 3 - \log 5$

$\log 6 - \log 3 = \log \dfrac{6}{3} = \log 2$

(iii) $\log_a m^n = n \log_a m$

or $\log m^n = n \log m$

Example:

$\log 2^3 = 3 \log 2$

$\log_a m \log_m n = \log_a n$

$\log_2 3 \log_3 5 = \log_3 5$

$\log_m n = \dfrac{\log_a n}{\log_a m}$

Example:

(iv) $\dfrac{\log_7 r}{\log_7 t} = \log_t r$

Note:

(i) $\log_a a = 1$

(ii) $\log_{10} 10 = 1$

(iii) $\log 10 = 1$

(iv) $\log_{10} 1 = 0$

(v) $\log 1 = 0$

(vi) $\log_m n = \dfrac{\log_a n}{\log_a m}$

This is called Change of Base Law

Ex # 3.5

Proof of Laws of Logarithm one by one

(i) $\log_a mn = \log_a m + \log_a n$

Proof:

$Let\ \ \log_a m = x\ \ and\ \ \log_a n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ a^y = n$

Now multiply these:

$a^x \times a^y = mn$

Or $mn = a^x \times a^y$

$mn = a^{x+y}$

Taking $\log_a$ on B.S

$\log_a mn = \log_a a^{x+y}$

$\log_a mn = (x+y)\log_a a$

$\log_a mn = (x+y)(1) \qquad \because \log_a a = 1$

$\log_a mn = x + y$

$\log_a mn = \log_a m + \log_a n$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof:

$Let\ \ \log_a m = x\ \ and\ \ \log_a n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ a^y = n$

Now Divide these:

$\frac{a^x}{a^y} = \frac{m}{n}$

Or

$\frac{m}{n} = \frac{a^x}{a^y}$

$\frac{m}{n} = a^{x-y}$

Taking $\log_a$ on B.S

$\log_a \frac{m}{n} = \log_a a^{x-y}$

$\log_a \frac{m}{n} = (x-y)\log_a a$

$\log_a \frac{m}{n} = (x-y)(1) \qquad \because \log_a a = 1$

$\log_a \frac{m}{n} = x - y$

$Hence\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

Ex # 3.5

(iii) $\log_a m^n = n\log_a m$

Proof:

$Let\ \ \log_a m = x$

In Exponential form:

$a^x = m$

Or

$m = a^x$

Taking power 'n' on B.S

$m^n = (a^x)^n$

$m^n = a^{nx}$

Taking $\log_a$ on B.S

$\log_a m^n = \log_a a^{nx}$

$\log_a m^n = nx\log_a a$

$\log_a m^n = nx(1) \qquad \because \log_a a = 1$

$\log_a m^n = nx$

$\log_a m^n = n\log_a m$

(iv) $\log_a m \log_m n = \log_a n$

Proof:

$Let\ \ \log_a m = x\ \ and\ \ \log_m n = y$

Write them in Exponential form:

$a^x = m\ \ and\ \ m^y = n$

Now multiply these:

$As\ a^{xy} = (a^x)^y$

$But\ \ (a^x)^y = m$

$So\ a^{xy} = (m)^y = n$

$So\ a^{xy} = n$

Taking $\log_a$ on B.S

$\log_a a^{xy} = \log_a n$

$(xy)\log_a a = \log_a n$

$xy(1) = \log_a n \qquad \because \log_a a = 1$

Now

$\log_a m \log_m n = \log_a n$

Example # 14 page # 90

$-1 + \log y$

Solution:

$= -1 + \log y$

$= -\log 10 + \log y$

$= \log 10^{-1} + \log y$

$= \log \frac{1}{10} + \log y$

$= \log 0.1 + \log y$

$= \log 0.1\, y$

Ex # 3.5

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Q1: Use logarithm properties to simplify the expression.

(i) $\log_7 \sqrt{7}$

Solution:

$\log_7 \sqrt{7}$

$Let\ x = \log_7 \sqrt{7}$

$x = \log_7 (7)^{\frac{1}{2}}$

$\boldsymbol{As\ \ \log_a m^n = n \log_a m}$

$x = \frac{1}{2} \log_7 7$

$x = \frac{1}{2}(1) \qquad \therefore \boldsymbol{\log_a a = 1}$

$x = \frac{1}{2}$

(ii) $\log_8 \frac{1}{2}$

Trick

Solution:

$\log_8 \frac{1}{2}$

$\log_8 \frac{1}{2}$

$Let\ \ \log_8 \frac{1}{2} = x$

In exponential form:

$8^x = \frac{1}{2}$

$(2^3)^x = 2^{-1}$

$2^{3x} = 2^{-1}$

Now

$3x = -1$

Divide B.S by 3, we get

$x = \frac{-1}{3}$

(iii) $\log_{10} \sqrt{1000}$

Solution:

$\log_{10} \sqrt{1000}$

$Let\ x = \log_{10} (10^3)^{\frac{1}{2}}$

$x = \log_{10} (10)^{\frac{3}{2}}$

$\boldsymbol{As\ \ \log_a m^n = n \log_a m}$

$x = \frac{3}{2} \log_{10} 10$

$x = \frac{3}{2}(1) \qquad \therefore \boldsymbol{\log_a a = 1}$

$x = \frac{3}{2}$

(iv) $\log_9 3 + \log_9 27$

Solution:

$\log_9 3 + \log_9 27$

$Let\ x = \log_9 3 + \log_9 27$

$\boldsymbol{As\ \ \log_a mn = \log_a m + \log_a n}$

$x = \log_9 3 \times 27$

$x = \log_9 81$

$x = \log_9 9^2$

$\boldsymbol{As\ \ \log_a m^n = n \log_a m}$

$x = 2 \log_9 9$

$x = 2(1) \qquad \therefore \boldsymbol{\log_a a = 1}$

$x = 2$

(v) $\log \frac{1}{(0.0035)^{-4}}$

Solution:

$\log \frac{1}{(0.0035)^{-4}}$

$Let\ \ x = \log \frac{1}{(0.0035)^{-4}}$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$x = \log 1 - \log(0.0035)^{-4}$

As $\boldsymbol{\log 1 = 0}$ and $\boldsymbol{\log_a m^n = n \log_a m}$

Thus

$x = 0 - (-4) \log 0.0035$

Here $Ch = -3$

And $M = .5441$

So

$x = 4(-3 + .5441)$

$x = 4(-2.4559)$

$x = -9.8236$

$R.W$
3.5×10^{-3}

Ex # 3.5

(vi) $\log 45$

Solution:

$\log 45$

$Let\ \ x = \log 45$

$x = \log 3 \times 3 \times 5$

$x = \log 3^2 \times 5$

$\boldsymbol{\log_a mn = \log_a m + \log_a n}$

and $\boldsymbol{\log_a m^n = n \log_a m}$

$x = 2\log 3 + \log 5$

$x = 2\log 3.00 + \log 5.00$

$x = 2(0 + .4771) + (0 + .6990)$

$x = 2(0.4771) + (0.6990)$

$x = 0.9542 + 0.6990$

$x = 1.6532$

Q2: Express each of the following as a single logarithm.

(i) $\boldsymbol{3\log 2 - 4\log 3}$

Solution:

$3\log 2 - 4\log 3$

$\boldsymbol{As\ \ \log_a m^n = n\log_a m}$

$3\log 2 - 4\log 3 = \log 2^2 - \log 3^4$

$3\log 2 - 4\log 3 = \log 8 - \log 81$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$3\log 2 - 4\log 3 = \log \frac{8}{81}$

(ii) $\boldsymbol{2\log 3 + 4\log 2 - 3}$

Solution:

$2\log 3 + 4\log 2 - 3$

$\boldsymbol{As\ \ \log_a m^n = n\log_a m}$

$2\log 3 + 4\log 2 - 3 = \log 3^2 + \log 2^4 - 3(1)$

$\boldsymbol{As\ \ \log 10 = 1}$

So

$2\log 3 + 4\log 2 - 3 = \log 9 + \log 16 - 3(\log 10)$

$\boldsymbol{As\ \ \log_a mn = \log_a m + \log_a n}$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 10^3$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 1000$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$2\log 3 + 4\log 2 - 3 = \log \frac{144}{1000}$

$2\log 3 + 4\log 2 - 3 = \log 0.144$

(iii) $\boldsymbol{\log 5 - 1}$

Solution:

$\log 5 - 1$

$\boldsymbol{As\ \ \log 10 = 1}$

$\log 5 - 1 = \log 5 - \log 10$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$\log 5 - 1 = \log \frac{5}{10}$

$\log 5 - 1 = \log 0.5$

(iv) $\boldsymbol{\frac{1}{2}\log x - 2\log 3y + 3\log z}$

Solution:

$\frac{1}{2}\log x - 2\log 3y + 3\log z$

$\boldsymbol{As\ \ \log_a m^n = n\log_a m}$

$= \log x^{\frac{1}{2}} - \log(3y)^2 + \log z^3$

$= \log \sqrt{x} - \log 9y^2 + \log z^3$

$\boldsymbol{As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$\boldsymbol{And\ \ \log_a mn = \log_a m + \log_a n}$

$\frac{1}{2}\log x - 2\log 3y + 3\log z = \log \frac{\sqrt{x}z^3}{9y^2}$

Q3: Find the value of $'a'$ from the following equations.

(i) $\boldsymbol{\log_2 6 + \log_2 7 = \log_2 a}$

Solution:

$\log_2 6 + \log_2 7 = \log_2 a$

$\boldsymbol{As\ \ \log_a mn = \log_a m + \log_a n}$

$\log_2 6 \times 7 = \log_2 a$

$\log_2 42 = \log_2 a$

Thus

$a = 42$

(ii) $\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

Solution:

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

$As\ \ \log_a mn = \log_a m + \log_a n$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{5 \times 8}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{40}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 20$

Thus

$a = 20$

(iii) $\frac{\log_7 r}{\log_7 t} = \log_a r$

Solution:

$\frac{\log_7 r}{\log_7 t} = \log_a r$

$As\ \ \log_m n = \frac{\log_a n}{\log_a m}$

$\log_t r = \log_a r$

Thus

$a = t$

(iv) $\log_6 25 - \log_6 5 = \log_6 a$

Solution:

$\log_6 25 - \log_6 5 = \log_6 a$

$As\ \ \log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_6 \frac{25}{5} = \log_6 a$

$\log_6 5 = \log_6 a$

Thus

$a = 5$

Q4: **Find $\log_2 3 . \log_3 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8$**

Solution:

$Let\ \ x = \log_2 3 . \log_3 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8$

$As\ \ \log_a m^n = n \log_a m$

So

$x = \log_2 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8$

$x = \log_2 5\ \ .\ \log_5 6 . \log_6 7 . \log_7 8$

$x = \log_2 6\ \ . \log_6 7 . \log_7 8$

$x = \log_2 7\ \ . \log_7 8$

$x = \log_2 8$

$x = \log_2 2^3$

$x = 3 \log_2 2$

$As\ \ \log_a a = 1$

$x = 3(1)$

$x = 3$

Ex # 3.6

Page # 93

Q1: **Simplify 3.81×43.4 with the help of logarithm.**

Solution:

(i) 3.81×43.4

Let $x = 3.81 \times 43.4$

Taking log on B.S

$\log x = \log 3.81 \times 43.4$

As $\log mn = \log m + \log n$

$\log x = \log 3.81 + \log 43.4$

$\log x = (0 + .5809) + (1 + .6375)$

$\log x = 0.5809 + 1.6375$

$\log x = 2.2184$

$\log 3.81$ $Ch = 0$ $M = .5809$
$\log 43.4$ $Ch = 1$ $M = .6375$

Taking anti − log on B.S

Anti − log $(\log x)$ = Anti − log 2.2184

x = Anti − log 2.2184

Here

Characteristics = 2

Mantissa = .2184

$1652 + 2$ $= 1654$

So

$x = 1.654 \times 10^2$

$x = 16.54$

Ex # 3.6

(ii) $\mathbf{73.42 \times 0.00462 \times 0.5143}$

Solution:

$73.42 \times 0.00462 \times 0.5143$

Let $x = 73.42 \times 0.00462 \times 0.5143$

Taking log on B.S

$\log x = 73.42 \times 0.00462 \times 0.5143$

As $\log mn = \log m + \log n$

$\log x = \log 73.42 + \log 0.00462 + \log 0.5143$

$\log x = (1 + .8658) + (-3 + .6646) + (-1 + .7113)$

$\log x = 1.8658 + (-2.3354) + (-0.2887)$

$\log x = 1.8658 - 2.3354 - 0.2887$

$\log x = -0.7583$

Add and Subtract -1

$\log x = -1 + 1 - 0.7583$

$\log x = -1 + .2417$

$\log x = \bar{1}\,.2417$

Taking $\text{anti} - \log$ on B. S

$\text{anti} - \log\ (\log x) = \text{anti} - \log \bar{1}\,.2417$

$x = \text{anti} - \log \bar{1}\,.2417$

Here

Characteristics $= -1$

Mantissa $= .2417$

So

$x = 1.745 \times 10^{-1}$

$x = 0.1745$

$\log 73.42$ $Ch = 1$ $8657 + 1$ $M = .8658$
$\log 0.00462$ $Ch = -3$ $M = .6646$
$\log 0.5143$ $Ch = -1$ $7110 + 3$ $M = .7113$

$1742 + 3$ $= 1745$

(iii) $\mathbf{\dfrac{784.6 \times 0.0431}{28.23}}$

Solution:

$\dfrac{784.6 \times 0.0431}{28.23}$

$Let\ x = \dfrac{784.6 \times 0.0431}{28.23}$

Taking log on B.S

$\log x = \log \dfrac{784.6 \times 0.0431}{28.23}$

$\boldsymbol{As\ \ \log \frac{m}{n} = \log m - \log n}$

$\log x = \log 784.6 \times 0.0431 - \log 28.23$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 784.6 + \log 0.0431 - \log 28.23$

Ex # 3.6

$\log x = (2 + .8946) + (-2 + .6345) + (1 + .4507)$

$\log x = 2.8946 + (-1.3655) + (1.4507)$

$\log x = 2.8946 - 1.3655 - 1.4507$

$\log x = 0\,.0784$

Taking anti − log on B. S

$\text{anti} - \log\ (\log x) = \text{anti} - \log 0\,.0784$

$x = \text{anti} - \log 0\,.0784$

Here

Characteristics $= 0$

Mantissa $= .0784$

So

$x = 1.198 \times 10^0$

$x = 1.198$

$\log 784.6$ $Ch = 2$ $8943 + 3$ $M = .8946$
$\log 0.0431$ $Ch = -2$ $M = .6345$
$\log 28.23$ $Ch = 1$ $4502 + 5$ $M = .4507$

$1197 + 1$ $= 1198$

(iv) $\dfrac{\mathbf{0.4932 \times 653.7}}{\mathbf{0.07213 \times 8456}}$

<u>Solution:</u>

$\dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

$Let\ x = \dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

Taking log on B.S

$\log x = \log \dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

$\boldsymbol{As\ \ \log \dfrac{m}{n} = \log m - \log n}$

$\log x = \log(0.4932 \times 653.7) - \log(0.07213 \times 8456)$

$\mathbf{As\ \log \boldsymbol{mn} = \log \boldsymbol{m} + \log \boldsymbol{n}}$

$\log x = \log 0.4932 + \log 653.7 - (\log 0.07213 + \log 8456)$

$\log x = \log 0.4932 + \log 653.7 - \log 0.07213 - \log 8456$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-0.3070) + (2.8154) - (-1.1419) - (3.9271)$

$\log x = -0.3070 + 2.8154 + 1.1419 - 3.9271$

$\log x = -0.2768$

$\log 0.4932$ $Ch = -1$ $6928 + 2$ $M = .6930$
$\log 653.7$ $Ch = 2$ $8149 + 5$ $M = .8154$
$\log 0.07213$ $Ch = -2$ $8579 + 2$ $M = .8581$
$\log 8456$ $Ch = 3$ $9269 + 3$ $M = .9272$

Ex # 3.6

Add and Subtract -1

$\log x = -1 + 1 - 0.2768$

$\log x = -1 + .7232$

$\log x = \bar{1}.7232$

Taking anti – log on B.S

$\text{anti} - \log\ (\log x) = \text{anti} - \log \bar{1}.7232$

$x = \text{anti} - \log \bar{1}..7232$

Here

Characteristics $= -1$

Mantissa $= .7232$

So

$x = 5.286 \times 10^{-1}$

$x = 0.5286$

$5284 + 2$ $= 5286$

(v) $\dfrac{(\mathbf{78.41})^3\sqrt{\mathbf{142.3}}}{\sqrt[4]{\mathbf{0.1562}}}$

<u>Solution:</u>

$\dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

$Let\ x = \dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

Taking log on B.S

$\log x = \log \dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

$\boldsymbol{As\ \ \log \dfrac{m}{n} = \log m - \log n}$

$\log x = \log(78.41)^3\sqrt{142.3} - \log \sqrt[4]{0.1562}$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log(78.41)^3 + \log\sqrt{142.3} - \log\sqrt[4]{0.1562}$

$\log x = \log(78.41)^3 + \log(142.3)^{\frac{1}{2}} - \log(0.1562)^{\frac{1}{4}}$

$\log x = 3\log 78.41 + \dfrac{1}{2}\log 142.3 - \dfrac{1}{4}\log 0.1562$

$\log x = 3\log(78.41) + \dfrac{1}{2}\log(142.3) - \dfrac{1}{4}\log(0.1562)$

$\log x = 3\,(1 + .8944) + \dfrac{1}{2}(\,2 + .1532) - \dfrac{1}{4}(-1 + .1937)$

$\log 78.41$ $Ch = 1$ $8943 + 1$ $M = .8944$
$\log 142.3$ $Ch = 2$ $1523 + 9$ $M = .1523$
$\log 0.1562$ $Ch = -1$ $1931 + 6$ $M = .1937$

$$\log x = 3\,(1.8944) + \frac{1}{2}(\,2.1532) - \frac{1}{4}(-0.8063)$$

Ex # 3.6

$\log x = 5.6832 + 1.0766 + 0.2016$

$\log x = 6.9614$

Taking anti − log on B.S

anti − log $(\log x) =$ anti − log 6.9614

$x =$ anti − log 6.9614

Here

Characteristics $= 6$

Mantissa $= .9614$

So

$x = 9.149 \times 10^6$

$x = 9149000$

$9141 + 8$ $= 9149$

Q2: Find the following if $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, $\log 7 = 0.8451$

(i) $\log 105$

Solution:

$\log 105$

$\log 105 = \log 3 \times 5 \times 7$

As $\log mn = \log m + \log n$

$\log 105 = \log 3 + \log 5 + \log 7$

$\log 105 = 0.4771 + 0.6990 + 0.8451$

$\log 105 = 2.0211$

(ii) $\log 108$

$\log 108$

$\log 108 = \log 2 \times 2 \times 3 \times 3 \times 3$

$\log 108 = \log 2^2 \times 3^3$

As $\log mn = \log m + \log n$

$\log 108 = \log 2^2 + \log 3^3$

As $\log_a m^n = n \log_a m$

$\log 108 = 2 \log 2 + 3 \log 3$

$\log 108 = 2(0.3010) + 3(0.4771)$

$\log 108 = 0.6020 + 1.4313$

$\log 108 = 2.0333$

(iii) $\log \sqrt[3]{72}$

Solution:

$\log \sqrt[3]{72}$

$\log \sqrt[3]{72} = \log(72)^{\frac{1}{3}}$

Review Ex # 3

As $\log_a m^n = n \log_a m$

$\log \sqrt[3]{72} = \frac{1}{3} \log 72$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2 \times 2 \times 2 \times 3 \times 3)$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2^3 \times 3^2)$

As $\log mn = \log m + \log n$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2^3 + log\, 3^2)$

$log \sqrt[3]{72} = \frac{1}{3}(3\, log\, 2 + 2\, log\, 3)$

$log \sqrt[3]{72} = \frac{1}{3}[3(0.3010) + 2(0.4771)]$

$log \sqrt[3]{72} = \frac{1}{3}[0.9030 + 0.9542]$

$log \sqrt[3]{72} = \frac{1}{3}[1.8572]$

$log \sqrt[3]{72} = 0.6191$

(iv) $\log 2.4$

Solution:

$\log 2.4$

$\log 2.4 = \log \frac{24}{10}$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log 2.4 = \log 24 - \log 10$

$\log 2.4 = \log 2 \times 2 \times 2 \times 3 - \log 10$

$\log 2.4 = \log 2^3 \times 3 - \log 10$

As $\log mn = \log m + \log n$

$\log 2.4 = \log 2^3 + \log 3 - \log 10$

As $\log_a m^n = n \log_a m$

$\log 2.4 = 3 \log 2 + \log 3 - \log 10$

$\log 2.4 = 3(0.3010) + 0.4771 - \log 10$

$\log 2.4 = 0.9030 + 0.4771 - 1 \;\therefore\; \mathbf{\log 10 = 1}$

$\log 2.4 = 1.3801 - 1$

$\log 2.4 = 0.3801$

Ex # 3.6

(v) $\log 0.0081$

Solution:

$\log 0.0081$

$$\log 0.0081 = \log \frac{81}{10000}$$

$$\log 0.0081 = \log \frac{3^4}{10^4}$$

$$\log 0.0081 = \log \left(\frac{3}{10}\right)^4$$

As $\log_a m^n = n \log_a m$

$$\log 0.0081 = 4 \log \frac{3}{10}$$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log 0.0081 = 4(\log 3 - \log 10)$

$\log 0.0081 = 4(0.4771 - 1) \quad \therefore \log 10 = 1$

$\log 0.0081 = 4(-0.5229)$

$$\log 0.0081 = -2.0916$$

REVIEW EXERCISE # 3

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Q2: Write 9473.2 in scientific notation.

9473.2

In scientific notation:

$$9.4732 \times 10^3$$

Q3: Write 5.4×10^6 in standard notation.

$$5.4 \times 10^6$$

In standard form:

5400000

Q4: Write in logarithm form: $3^{-3} = \frac{1}{27}$

$$3^{-3} = \frac{1}{27}$$

In logarithm form:

$$\log_3 \frac{1}{27} = -3$$

Review Ex # 3

Q5: Write in exponential form: $\log_5 1 = 0$

$$\log_5 1 = 0$$

In exponential form:

$$5^0 = 1$$

Q6: Solve for x: $\log_4 16 = x$

$$\log_4 16 = x$$

In exponential form:

$$4^x = 16$$

$$4^x = 4^2$$

So

$$x = 2$$

Q7: Find the characteristic of the common logarithm 0.0083.

0.0083

In scientific notation:

$$8.3 \times 10^{-3}$$

So Characteristics -3

Q8: Find $\log 12.4$

$$\log 12.4$$

In Scientific form:

$$1.24 \times 10^1$$

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

Mantissa $= .0934$

Hence $\log 12.4 = 0.0934$

Q9: Find the value of $'a'$,

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

Solution:

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

As $\log_a mn = \log_a m + \log_a n$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} \frac{9 \times 2}{3}$$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 3 \times 2$

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 6$

Thus $3a = 6$

$$a = \frac{6}{2}$$

$a = 3$

Q10 $\dfrac{\mathbf{(63.28)^3(0.00843)^2(0.4623)}}{\mathbf{(412.3)(2.184)^5}}$

Solution:

$$\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$Let\ x=\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

Taking log on B.S

$$\log x=\log\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$As\ \ \log\frac{m}{n}=\log m-\log n$$

$$\log x=\log((63.28)^3(0.00843)^2(0.4623))-\log((412.3)(2.184)^5)$$

As $\log mn=\log m+\log n$

$$\log x=\log(63.28)^3+\log(0.00843)^2+\log 0.4623-(\log 412.3+\log(2.184)^5)$$

$$\log x=3\log 63.28+2\log 0.00843+\log 0.4623-(\log 412.3+5\log 2.184)$$

$$\log x=3\log 63.28+2\log 0.00843+\log 0.4623-\log 412.3-5\log 2.184$$

$$\log x=3(1+.8012)+2(-3+.9258)+(-1+.6649)-(2+.6152)-5(0+.3393)$$

$$\log x=3(1.8012)+2(-2.0742)+(-0.3351)-(2.6152)-5(0.3393)$$

$$\log x=5.4036-4.1484-0.3351-2.6152-1.6965$$

$$\log x=-3.3916$$

Add and Subtract -4

$\log x=-4+4-3.3916$

$\log x=-4+.6084$

$\log x=\bar{4}.6084$

Taking anti $-$ log on B. S

$\text{anti}-\log\ (\log x)=\text{anti}-\log\bar{4}.6084$

$x=\text{anti}-\log\bar{4}.6084$

Here

Characteristics $=-4$

Mantissa $=.6084$

So

$x=4.059\times10^{-4}$

$x=0.000405$

$\log 63.28$ $Ch=1$ $8007+5$ $M=.8012$
$\log 0.00843$ $Ch=-3$ $M=.9258$
$\log 0.4623$ $Ch=-1$ $6646+3$ $M=.6649$
$\log 412.3$ $Ch=2$ $6149+3$ $M=.6152$
$\log 2.184$ $Ch=0$ $3385+8$ $M=.3393$

$4055+4$
$=4059$

Chapter # 4

UNIT # 4

ALGEBRAIC EXPRESSIONS & ALGEBRAIC FORMULAS

Ex # 4.1

Algebraic Expressions

When variables and constants are connected by algebraic operations like addition, subtraction, multiplication, division, root extraction & rising integral or fractional powers is called algebraic expressions.

Variable:

A quantity that value may change within the context of problem. It is unknown value.

Normally, we use English letters for variables

Example:

a, d, e, x, y, z

Constant:

A quantity that value doesn't change. It is a fixed value.

Example:

4, 6, 267, 983384

Constant

جس کی value تبدیل نہیں ہوتی یعنی 1,2,3,9,22

Variable

جس کی value تبدیل ہوتی یعنی a,b,c,x,y,z

For Addition and Subtraction and other important terminologies

Visit this video:

https://youtu.be/4jFH9OMmjXI

Polynomial

The algebraic expression in which powers of variables are whole numbers is called polynomial.

Rational Expression:

An expression of form of $\frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials and $q(x) \neq 0$.

Example:

$$\frac{x^2 - 6x + 1}{x + 9}$$

$$\frac{4x^2 + 10x + 11}{5}$$

Note:

Every polynomial $p(x)$ is a rational expression but every rational expression need not to be a polynomial.

Irrational Expression:

An expression which cannot be written in the form of $\frac{p(x)}{q(x)}$

Term

Different parts of an algebraic expression joined by the operations of addition and subtraction are called term.

Example

$3x^3 + 5\sqrt{x} - 7$. The terms are $\mathbf{3x^3}$, $\mathbf{5\sqrt{x}}$, $\mathbf{-7}$

Rules to express a rational expression in its lowest term

$$Let\ \frac{p(x)}{q(x)}$$

Step 1: Factorize both the polynomial in the numerator and denominator.

Step 2: cancel the common factors between them.

Example # 9

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Chapter # 4

Ex # 4.1

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Q1: Which of the following expressions are polynomials?

(i) $\mathbf{1-5y+8y^2+6y^3}$

Ans: Polynomial and also Rational

(ii) $\dfrac{5}{x^2}+\dfrac{3}{4x+1}$

Ans: Non-Polynomial but Rational

(iii) $\dfrac{\sqrt{x}}{6x-1}$

Ans: Non-Polynomial but Irrational

Q2: Which of the following rational expressions are in their lowest terms?

(i) $\dfrac{5y^2-5}{y-1}$

Solution:

$$\frac{5y^2--5}{y-1}$$

$$\frac{5y^2-5}{y-1}=\frac{5(y^2-1)}{y-1}$$

$$\frac{5y^2-5}{y-1}=\frac{5(y+1)(y-1)}{y-1}$$

$$\frac{5y^2-5}{y-1}=5(y+1)$$

So it is **Not** in Lowest Term:

(ii) $\dfrac{x^2-9}{x-2}$

Solution:

$$\frac{x^2-9}{x-2}$$

$$\frac{x^2-9}{x-2}=\frac{(x+3)(x-3)}{x-2}$$

We can't solve it more

So it is in Lowest Term

(iii) $\dfrac{x+y}{x^2-y^2}$

Solution:

$$\frac{x+y}{x^2-y^2}$$

$$\frac{x+y}{x^2-y^2}=\frac{x+y}{(x+y)(x-y)}$$

$$\frac{x+y}{x^2-y^2}=\frac{1}{x-y}$$

So it is **Not** in Lowest Term:

Q3: Reduce the following rational expression to their lowest term:

(i) $\dfrac{x-5}{x^2-5x}$

Solution:

$$\frac{x-5}{x^2-5x}$$

$$\frac{x-5}{x^2-5x}=\frac{x-5}{x(x-5)}$$

$$\frac{x-5}{x^2-5x}=\frac{1}{x}$$

(ii) $\dfrac{t^3(t-3)}{(t-3)(t+5)}$

Solution:

$$\frac{t^3(t-3)}{(t-3)(t+5)}$$

$$\frac{t^3(t-3)}{(t-3)(t+5)}=\frac{t^3}{(t+5)}$$

(iii) $\dfrac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$

Solution:

$$\frac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$$

Ans: It cannot be reduced further

Chapter # 4

Ex # 4.1

(iv) $\frac{2a+6}{a^2-9}$

Solution:

$\frac{2a+6}{a^2-9}$

$\frac{2a+6}{a^2-9} = \frac{2(a+3)}{(a+3)(a-3)}$

$\frac{2a+6}{a^2-9} = \frac{2}{(a-3)}$

Q4: Add the following rational expressions:

(i) $4x^2-5x-10, \quad 2x^2+5x+10$

Solution:

$4x^2-5x-10, \quad 2x^2+5x+10$

Now

$(4x^2-5x-10)+(2x^2+5x+10)$

$= 4x^2-5x-10+2x^2+5x+10$

Write the like term

$= 4x^2+2x^2-5x+5x-10+10$

$= 6x^2$

(ii) $\frac{y+9}{y^2+3}, \quad \frac{-7y+7}{y^2+3}$

Solution:

$\frac{y+9}{y^2+3}, \quad \frac{-7y+7}{y^2+3}$

$= \frac{y+9}{y^2+3} + \frac{-7y+7}{y^2+3}$

$= \frac{(y+9)+(-7y+7)}{y^2+3}$

$= \frac{y+9-7y+7}{y^2+3}$

$= \frac{y-7y+9+7}{y^2+3}$

$= \frac{-6y+16}{y^2+3}$

Ex # 4.1

(iii) $\frac{y}{y+4}, \quad \frac{2y}{y-4}$

Solution:

$\frac{y}{y+4}, \quad \frac{2y}{y-4}$

$= \frac{y}{y+4} + \frac{2y}{y-4}$

$= \frac{y(y-4)+2y(y+4)}{(y+4)(y-4)}$

$= \frac{y^2-4y+2y^2+8y}{(y+4)(y-4)}$

$= \frac{y^2+2y^2-4y++8y}{x^2-4^2}$

$= \frac{3y^2+4y}{x^2-16}$

(iv) $\frac{t}{t^2-25}, \quad \frac{3t}{t+5}$

Solution:

$\frac{t}{t^2-25}, \quad \frac{3t}{t+5}$

$\frac{t}{t^2-25} + \frac{3t}{t+5}$

$\frac{t}{(t+5)(t-5)} + \frac{3t}{t+5}$

$\frac{t+3t(t-5)}{(t+5)(t-5)}$

$\frac{t+3t^2-15t}{t^2-5^2}$

$\frac{3t^2+t-15t}{t^2-25}$

$\frac{3t^2-14t}{t^2-25}$

Chapter # 4

Ex # 4.1

Q5: Subtract the first expression from the second in the following.

(i) $\boldsymbol{y^2 + 4y - 15}\,,\quad \boldsymbol{8y^2 + 2}$

Solution:

$y^2 + 4y - 15\,,\quad 8y^2 + 2$

$= (8y^2 + 2) - (y^2 + 4y - 15)$

$= 8y^2 + 2 - y^2 - 4y + 15$

$= 8y^2 - y^2 - 4y + 2 + 15$

$= 7y^2 - 4y + 17$

(ii) $\boldsymbol{\dfrac{8x^2 - 7}{x^2 + 1}}\,,\quad \boldsymbol{\dfrac{8x^2 + 7}{x^2 + 1}}$

Solution:

$\dfrac{8x^2 - 7}{x^2 + 1}\,,\quad \dfrac{8x^2 + 7}{x^2 + 1}$

$= \dfrac{8x^2 + 7}{x^2 + 1} - \dfrac{8x^2 - 7}{x^2 + 1}$

$= \dfrac{(8x^2 + 7) - (8x^2 - 7)}{x^2 + 1}$

$= \dfrac{8x^2 + 7 - 8x^2 + 7}{x^2 + 1}$

$= \dfrac{8x^2 - 8x^2 + 7 + 7}{x^2 + 1}$

$= \dfrac{14}{x^2 + 1}$

(iii) $\boldsymbol{\dfrac{1}{a - 3}}\,,\quad \boldsymbol{\dfrac{2a}{a^2 - 9}}$

Solution:

$\dfrac{1}{a - 3}\,,\quad \dfrac{2a}{a^2 - 9}$

$= \dfrac{2a}{a^2 - 9} - \dfrac{1}{a - 3}$

$= \dfrac{2a}{(a + 3)(a - 3)} - \dfrac{1}{a - 3}$

$= \dfrac{2a - 1(a + 3)}{(a + 3)(a - 3)}$

$= \dfrac{2a - a - 3}{(a + 3)(a - 3)}$

Ex # 4.1

$= \dfrac{a - 3}{(a + 3)(a - 3)}$

$= \dfrac{1}{(a + 3)}$

(iv) $\boldsymbol{\dfrac{x}{3x - 6}}\,,\quad \boldsymbol{\dfrac{x + 2}{x - 2}}$

Solution:

$\dfrac{x}{3x - 6}\,,\quad \dfrac{x + 2}{x - 2}$

$= \dfrac{x + 2}{x - 2} - \dfrac{x}{3x - 6}$

$= \dfrac{x + 2}{x - 2} - \dfrac{x}{3(x - 2)}$

$= \dfrac{3(x + 2) - x}{3(x - 2)}$

$= \dfrac{3x + 6 - x}{3(x - 2)}$

$= \dfrac{3x - x + 6}{3(x - 2)}$

$= \dfrac{2x + 6}{3(x - 2)}$

$= \dfrac{2(x + 3)}{3(x - 2)}$

Q6: Simplify the following.

(i) $\boldsymbol{\dfrac{2x}{6x - 9} \cdot \dfrac{4x - 6}{x^2 + x}}$

Solution:

$\dfrac{2x}{6x - 9} \cdot \dfrac{4x - 6}{x^2 + x}$

$= \dfrac{2x}{3(2x - 3)} \cdot \dfrac{2(2x - 3)}{x(x + 1)}$

$= \dfrac{2}{3} \cdot \dfrac{2}{(x + 1)}$

$= \dfrac{4}{3(x + 1)}$

Chapter # 4

Ex # 4.1

(ii) $\dfrac{\boldsymbol{x+4}}{\boldsymbol{3-x}} \cdot \dfrac{\boldsymbol{x^2-9}}{\boldsymbol{x^2-16}}$

Solution:

$$\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$$
$$= \frac{x+4}{-x+3} \cdot \frac{x^2-3^2}{x^2-4^2}$$
$$= \frac{x+4}{-(x-3)} \cdot \frac{(x+3)(x-3)}{(x+4)(x-4)}$$
$$= \frac{1}{-1} \cdot \frac{(x+3)}{(x-4)}$$
$$= \frac{1(x+3)}{-1(x-4)}$$
$$= \frac{x+3}{-x+4}$$
$$= \frac{x+3}{4-x}$$

(iii) $\dfrac{\boldsymbol{3x-15}}{\boldsymbol{2x+6}} \cdot \dfrac{\boldsymbol{x^2-9}}{\boldsymbol{x^2-25}}$

Solution:

$$\frac{3x-15}{2x+6} \cdot \frac{x^2-9}{x^2-25}$$
$$= \frac{3(x-5)}{2(x+3)} \cdot \frac{(x+3)(x-3)}{(x+5)(x-5)}$$
$$= \frac{3}{2} \cdot \frac{(x-3)}{(x-5)}$$
$$= \frac{3(x-3)}{2(x-5)}$$

Q7: Simplify the following.

(i) $\dfrac{\boldsymbol{2y-10}}{\boldsymbol{3y}} \div (\boldsymbol{y-5})$

Solution:

$$\frac{2y-10}{3y} \div (y-5)$$
$$= \frac{2(y-5)}{3y} \times \frac{1}{y-5}$$
$$= \frac{2}{3y}$$

Ex # 4.1

(ii) $\dfrac{\boldsymbol{p}}{\boldsymbol{q}} \div \dfrac{\boldsymbol{r}}{\boldsymbol{q}} \cdot \dfrac{\boldsymbol{p}}{\boldsymbol{q}}$

Solution:

$$\frac{p}{q} \div \frac{r}{q} \cdot \frac{p}{q}$$
$$= \frac{p}{q} \cdot \frac{q}{r} \cdot \frac{p}{q}$$
$$= \frac{p}{q} \cdot \frac{1}{r} \cdot \frac{p}{1}$$
$$= \frac{p^2}{qr}$$

(iii) $\dfrac{\boldsymbol{a^2-9}}{\boldsymbol{(a-6)(a+4)}} \div \dfrac{\boldsymbol{a-3}}{\boldsymbol{a-6}}$

Solution:

$$\frac{a^2-9}{(a-6)(a+4)} \div \frac{a-3}{a-6}$$
$$= \frac{(a+3)(a-3)}{(a-6)(a+4)} \times \frac{a-6}{a-3}$$
$$= \frac{(a+3)}{(a+4)}$$
$$= \frac{a+3}{a+4}$$

Ex # 4.2

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Q1: Evaluate the following when $\boldsymbol{a=3}$, $\boldsymbol{b=-1}$, $\boldsymbol{c=2}$.

(i) $\boldsymbol{5a-10}$

Solution:

$5a-10$

$5a-10 = 5(3)-10$

$5a-10 = 15-10$

$5a-10 = 5$

Chapter # 4

Ex # 4.2

(ii) $\mathbf{3b+5c}$

Solution:

$3b+5c$

$3b+5c=3(-1)+5(2)$

$3b+5c=-3+10$

$3b+5c=7$

(iii) $\mathbf{2a-3b+2c}$

Solution:

$2a-3b+2c$

$2a-3b+2c=2(3)-3(-1)+2(2)$

$2a-3b+2c=6+3+4$

$2a-3b+2c=13$

Q2: Evaluate the following for $x=-5$ and $y=2$.

(i) $\mathbf{7-3xy}$

Solution:

$7-3xy$

$7-3xy=7-3(-5)(2)$

$7-3xy=7-3(-10)$

$7-3xy=7+30$

$7-3xy=37$

(ii) $\mathbf{x^2+xy+y^2}$

Solution:

x^2+xy+y^2

$x^2+xy+y^2=(-5)^2+(-5)(2)+(2)^2$

$x^2+xy+y^2=25+(-10)+4$

$x^2+xy+y^2=25-10+4$

$x^2+xy+y^2=15+4$

$x^2+xy+y^2=19$

(iii) $\mathbf{(3x)^2-(4y)^2}$

Solution:

$(3x)^2-(4y)^2$

$(3x)^2-(4y)^2=[3(-5)]^2-[4(2)]^2$

$(3x)^2-(4y)^2=[-15]^2-[8]^2$

$(3x)^2-(4y)^2=225-64$

$(3x)^2-(4y)^2=161$

Ex # 4.2

Q3: Evaluate the following when $k=-2$, $l=3$, $m=4$.

(i) $\mathbf{k^2(2l-3m)}$

Solution:

$k^2(2l-3m)$

$k^2(2l-3m)=(-2)^2[2(3)-3(4)]$

$k^2(2l-3m)=4(6-12)$

$k^2(2l-3m)=4(-6)$

$k^2(2l-3m)=-24$

(ii) $\mathbf{5m\sqrt{k^2+l^2}}$

Solution:

$5m\sqrt{k^2+l^2}$

$5m\sqrt{k^2+l^2}=5(4)\sqrt{(-2)^2+(3)^2}$

$5m\sqrt{k^2+l^2}=20\sqrt{4+9}$

$5m\sqrt{k^2+l^2}=20\sqrt{13}$

(iii) $\mathbf{\dfrac{k+l+m}{k^2+l^2+m^2}}$

Solution:

$\dfrac{k+l+m}{k^2+l^2+m^2}$

Put the values

$\dfrac{k+l+m}{k^2+l^2+m^2}=\dfrac{(-2)+(3)+(4)}{(-2)^2+(3)^2+(4)^2}$

$\dfrac{k+l+m}{k^2+l^2+m^2}=\dfrac{-2+3+4}{4+9+16}$

$\dfrac{k+l+m}{k^2+l^2+m^2}=\dfrac{1+4}{13+16}$

$\dfrac{k+l+m}{k^2+l^2+m^2}=\dfrac{5}{29}$

Chapter # 4

Ex # 4.2

Q4: $\boldsymbol{Evaluate\ \frac{a+1}{4a^2+1}\ when}$ $\boldsymbol{a=\frac{1}{2}}$ **and** $\boldsymbol{a=-\frac{1}{2}}$.

Solution:

$\boldsymbol{For\ a=-\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}+1}{4\left(\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{3}{2}}{1+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{3}{2}}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{3}{2}\div 2$$

$$\frac{a+1}{4a^2+1}=\frac{3}{2}\times\frac{1}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{3}{4}$$

$\boldsymbol{For\ a=-\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{-\frac{1}{2}+1}{4\left(-\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{-1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}}{1+1}$$

Ex # 4.2

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{1}{2}\div 2$$

$$\frac{a+1}{4a^2+1}=\frac{1}{2}\times\frac{1}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{1}{4}$$

Q5: $\boldsymbol{If\ a=9,\quad b=12, c=15\ and}$ $\boldsymbol{s=\frac{a+b+c}{2}}$.

$\boldsymbol{Find\ the\ \ value\ of\ \sqrt{s(s-a)(s-b)(s-c)}}$

Solution:

Given:

$$a=9, b=12, c=15\ and\ s=\frac{a+b+c}{2}$$

To Find:

$$\sqrt{s(s-a)(s-b)(s-c)}=?$$

First we find:

$$s=\frac{a+b+c}{2}$$

Put the values:

$$s=\frac{a+b+c}{2}$$

$$s=\frac{9+12+15}{2}$$

$$s=\frac{36}{2}$$

$$s=18$$

Now

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18(18-9)(18-12)(18-15)}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18(9)(6)(3)}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{2\times 9\times 9\times 2\times 3\times 3}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9\times 9\times 2\times 2\times 3\times 3}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9^2\times 2^2\times 3^2}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=9\times 2\times 3$$

$$\sqrt{s(s-a)(s-b)(s-c)}=9\times 6$$

$$\sqrt{s(s-a)(s-b)(s-c)}=54$$

Chapter # 4

Ex # 4.3

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $a^2-b^2 = (a+b)(a-b)$
4. $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$ **Q2, Q3(i)**
5. $(a+b)^2-(a-b)^2 = 4ab$ **Q2, Q3(ii)**
6. $(x+y)^2+(x-y)^2 = 2(x^2+y^2)$ **Q1, Q5**
7. $(x+y)^2-(x-y)^2 = 4xy$ **Q1, Q4, Q5**
8. $(u+v)^2-(u-v)^2 = 4uv$ **Q6**

Ex # 4.3

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Q1: Find the value of x^2+y^2 and xy, when:

(i) $\boldsymbol{x+y=8,\quad x-y=3}$

Solution:

$x+y=8,\quad x-y=3$

To Find:

x^2+y^2 =? and xy =?

$\underline{\boldsymbol{x^2+y^2}}$

As we have

$\boldsymbol{(x+y)^2+(x-y)^2=2(x^2+y^2)}$

Put the values

$(8)^2+(3)^2=2(x^2+y^2)$

$64+9=2(x^2+y^2)$

$73=2(x^2+y^2)$

Divide B.S by 2

$\frac{73}{2}=\frac{2(x^2+y^2)}{2}$

$\frac{73}{2}=x^2+y^2$

$x^2+y^2=\frac{73}{2}$

$\underline{\boldsymbol{xy}}$

Also we have

$\boldsymbol{(x+y)^2-(x-y)^2=4xy}$

Put the values

$(8)^2-(3)^2=4xy$

$64-9=4xy$

$55=4xy$

Divide B.S by 4

$\frac{55}{4}=\frac{4xy}{4}$

$\frac{55}{4}=xy$

$xy=\frac{55}{4}$

Ex # 4.3

(ii) $\boldsymbol{x+y=10,\quad x-y=7}$

Solution:

$x+y=10,\quad x-y=7$

To Find:

x^2+y^2 =? And xy =?

$\underline{\boldsymbol{x^2+y^2}}$

As we have

$\boldsymbol{(x+y)^2+(x-y)^2=2(x^2+y^2)}$

Put the values

$(10)^2+(7)^2=2(x^2+y^2)$

$100+49=2(x^2+y^2)$

$149=2(x^2+y^2)$

Divide B.S by 2

$\frac{149}{2}=\frac{2(x^2+y^2)}{2}$

$\frac{149}{2}=x^2+y^2$

$x^2+y^2=\frac{149}{2}$

$\underline{\boldsymbol{xy}}$

Also we have

$\boldsymbol{(x+y)^2-(x-y)^2=4xy}$

Put the values

$(10)^2-(7)^2=4xy$

$100-49=4xy$

$51=4xy$

Divide B.S by 4

$\frac{51}{4}=\frac{4xy}{4}$

$\frac{51}{4}=xy$

$xy=\frac{51}{4}$

(iii) $\boldsymbol{x+y=11,\quad x-y=5}$

Solution:

$x+y=11,\quad x-y=5$

To Find:

x^2+y^2 =? and xy =?

$\underline{\boldsymbol{x^2+y^2}}$

As we have

$\boldsymbol{(x+y)^2+(x-y)^2=2(x^2+y^2)}$

Put the values

$(11)^2+(5)^2=2(x^2+y^2)$

Chapter # 4

Ex # 4.3

$121+25=2(x^2+y^2)$
$146=2(x^2+y^2)$
Divide B.S by 2
$\frac{146}{2}=\frac{2(x^2+y^2)}{2}$
$73=x^2+y^2$
$x^2+y^2=73$

<u>xy</u>

Also we have
$$(x+y)^2-(x-y)^2=4xy$$
Put the values
$(11)^2-(5)^2=4xy$
$121-25=4xy$
$96=4xy$
Divide B.S by 4
$\frac{96}{4}=\frac{4xy}{4}$
$24=xy$
$xy=24$

(iv) $\boldsymbol{x+y=7,\quad x-y=4}$

<u>Solution:</u>
$x+y=7,\quad x-y=4$
To Find:
$x^2+y^2=?$ and $xy=?$

<u>x^2+y^2</u>

As we have
$$(x+y)^2+(x-y)^2=2(x^2+y^2)$$
Put the values
$(7)^2+(4)^2=2(x^2+y^2)$
$49+16=2(x^2+y^2)$
$65=2(x^2+y^2)$
Divide B.S by 2
$\frac{65}{2}=\frac{2(x^2+y^2)}{2}$
$\frac{65}{2}=x^2+y^2$
$x^2+y^2=\frac{65}{2}$

<u>xy</u>

Also we have
$$(x+y)^2-(x-y)^2=4xy$$
Put the values
$(7)^2-(4)^2=4xy$

Ex # 4.3

$49-16=4xy$
$33=4xy$
Divide B.S by 4
$\frac{33}{4}=\frac{4xy}{4}$
$\frac{33}{4}=xy$
$xy=\frac{33}{4}$

Q2: Find the value of $\boldsymbol{a^2+b^2}$ and $\boldsymbol{ab}$, when

(i) $\boldsymbol{a+b=7,\qquad a-b=3}$

<u>Solution:</u>
$a+b=7$ and $a-b=3$
To Find:
$a^2+b^2=?$ and $ab=?$

<u>a^2+b^2</u>

As we have
$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$
Put the values
$(7)^2+(3)^2=2(a^2+b^2)$
$49+9=2(a^2+b^2)$
$58=2(a^2+b^2)$
Divide B.S by 2
$\frac{58}{2}=\frac{2(a^2+b^2)}{2}$
$29=a^2+b^2$
$a^2+b^2=29$

<u>ab</u>

Also we have
$$(a+b)^2-(a-b)^2=4ab$$
Put the values
$(7)^2-(3)^2=4ab$
$49-9=4ab$
$40=4ab$
Divide B.S by 4
$\frac{40}{4}=\frac{4ab}{4}$
$10=ab$
$ab=10$

Chapter # 4

Ex # 4.3

Q2: (ii) **Find the value of a^2+b^2 and ab, when $a+b=9$, $a-b=1$.**

Solution:

$a+b=9$ and $a-b=1$

To Find:

$a^2+b^2=?$ and $ab=?$

$\underline{a^2+b^2}$

As we have

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Put the values

$(9)^2+(1)^2=2(a^2+b^2)$

$81+1=2(a^2+b^2)$

$82=2(a^2+b^2)$

Divide B.S by 2

$\frac{82}{2}=\frac{2(a^2+b^2)}{2}$

$41=a^2+b^2$

$a^2+b^2=41$

$\underline{ab}$

Also we have

$$(a+b)^2-(a-b)^2=4ab$$

Put the values

$(9)^2-(1)^2=4ab$

$81-1=4ab$

$80=4ab$

Divide B.S by 4

$\frac{80}{4}=\frac{4ab}{4}$

$20=ab$

$ab=20$

Q3: (i) **If $a+b=10$, $a-b=6$, then find the value of a^2+b^2.**

Solution:

$a+b=10$ and $a-b=6$

To Find:

$a^2+b^2=?$

As we have

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Put the values

$(10)^2+(6)^2=2(a^2+b^2)$

$100+36=2(a^2+b^2)$

$136=2(a^2+b^2)$

Ex # 4.3

Divide B.S by 2

$\frac{136}{2}=\frac{2(a^2+b^2)}{2}$

$68=a^2+b^2$

$a^2+b^2=68$

Q3: (ii) **If $a+b=5$, $a-b=\sqrt{17}$, then find the value of ab.**

Solution:

$a+b=5$ and $a-b=\sqrt{17}$

To Find:

$$ab=?$$

Also we have

$$(a+b)^2-(a-b)^2=4ab$$

Put the values

$(5)^2-\left(\sqrt{17}\right)^2=4ab$

$25-17=4ab$

$8=4ab$

Divide B.S by 4

$\frac{8}{4}=\frac{4ab}{4}$

$2=ab$

$ab=2$

Q4: **Find the value of $4xy$ when $x+y=17$, $x-y=5$.**

Solution:

$$x+y=17,\quad x-y=5$$

To find:

$$4xy=?$$

Also we have

$$(x+y)^2-(x-y)^2=4xy$$

Put the values

$(17)^2-(5)^2=4xy$

$289-25=4xy$

$264=4xy$

OR

$4xy=264$

Chapter # 4

Ex # 4.3

Q5: **If $+y = 11$ *and* $x - y = 3$,**
find $8xy(x^2 + y^2)$.

Solution:

$x + y = 11, \quad x - y = 3$

To Find:

$8xy(x^2 + y^2) = ?$

As we have

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

Put the values

$(11)^2 + (3)^2 = 2(x^2 + y^2)$

$121 + 9 = 2(x^2 + y^2)$

$130 = 2(x^2 + y^2)$

$2(x^2 + y^2) = 130 \quad --equ(i)$

Also we have

$$(x + y)^2 - (x - y)^2 = 4xy$$

Put the values

$(11)^2 - (3)^2 = 4xy$

$121 - 9 = 4xy$

$112 = 4xy$

$4xy = 112 \quad --equ(ii)$

Multiply equ (i) and (ii)

$2(x^2 + y^2) \times 4xy = 130 \times 112$

$8xy(x^2 + y^2) = 14560$

Q6: **If $u + v = 7$ and $uv = 12$, find $u - v$.**

Solution:

$u + v = 7, \ uv = 12$

To Find:

$u - v = ?$

As we know that

$$(u + v)^2 - (u - v)^2 = 4uv$$

Put the values

$(7)^2 - (u - v)^2 = 4(12)$

$49 - (u - v)^2 = 48$

$-(u - v)^2 = 48 - 49$

$-(u - v)^2 = -1$

$(u - v)^2 = 1$

Taking square root on B.S

$\sqrt{(u - v)^2} = \sqrt{1}$

$u - v = \pm 1$

Ex # 4.4

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
Q1, Q2, Q3
2. $2(x^2 + y^2 + z^2 - xy - yz - zx) =$
$(x - y)^2 + (y - z)^2 + (z - x)^2$ **Q4, Q5**
3. $2(a^2 + b^2 + c^2 - ab - bc - ca) =$
$(a - b)^2 + (b - c)^2 + (c - a)^2$ **Q6**

Ex # 4.4

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Q1: **Find the values of $a^2 + b^2 + c^2$, when**

(i) **$a + b + c = 5$ and $ab + bc + ca = -4$**

Solution:

$a + b + c = 5$ and $ab + bc + ca = -4$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-4)$

$25 = a^2 + b^2 + c^2 - 8$

$25 + 8 = a^2 + b^2 + c^2$

$33 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 33$

(ii) **$a + b + c = 5$ and $ab + bc + ca = -2$**

Solution:

$a + b + c = 5$ and $ab + bc + ca = -2$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-2)$

$25 = a^2 + b^2 + c^2 - 4$

$25 + 4 = a^2 + b^2 + c^2$

$29 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 29$

Chapter # 4

Ex # 4.4

Q2: Find the values of $a+b+c$, when

(i) $a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

Solution:

$a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

To Find:

$a+b+c=?$

As we know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

Put the values

$(a+b+c)^2=38+2(-1)$

$(a+b+c)^2=38-2$

$(a+b+c)^2=36$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{36}$

$a+b+c=6$

(ii) $a^2+b^2+c^2=10$ and $ab+bc+ca=11$

Solution:

$a^2+b^2+c^2=10$ and $ab+bc+ca=11$

To Find:

$$a+b+c=?$$

As we know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

Put the values

$(a+b+c)^2=10+2(11)$

$(a+b+c)^2=10+22$

$(a+b+c)^2=32$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{32}$

$a+b+c=\sqrt{16\times 2}$

$a+b+c=\sqrt{16}\times\sqrt{2}$

$a+b+c=4\sqrt{2}$

Q3: Find the values of $ab+bc+ca$, when

(i) $a^2+b^2+c^2=56$ and $a+b+c=12$

Solution:

$a^2+b^2+c^2=56$ and $a+b+c=12$

To Find:

$$ab+bc+ca=?$$

As we know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

Ex # 4.4

Put the values

$(12)^2=56+2(ab+bc+ca)$

$144=56+2(ab+bc+ca)$

Subtract 56 from B.S

$144-56=56-56+2(ab+bc+ca)$

$88=2(ab+bc+ca)$

Divide B.S by 2

$\frac{88}{2}=\frac{2(ab+bc+ca)}{2}$

$44=ab+bc+ca$

$ab+bc+ca=44$

(ii) $a^2+b^2+c^2=12$ and $a+b+c=5$

Solution:

$a^2+b^2+c^2=12$ and $a+b+c=5$

To Find:

$$ab+bc+ca=?$$

As we know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

Put the values

$(5)^2=12+2(ab+bc+ca)$

$25=12+2(ab+bc+ca)$

Subtract 12 from B.S

$25-12=12-12+2(ab+bc+ca)$

$13=2(ab+bc+ca)$

Divide B.S by 2

$\frac{13}{2}=\frac{2(ab+bc+ca)}{2}$

$\frac{13}{2}=ab+bc+ca$

$ab+bc+ca=\frac{13}{2}$

Chapter # 4

Ex # 4.4

Q #4 **Prove that $x^2+y^2+y^2-xy-yz-zx=\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$**

Solution:

$x^2+y^2+y^2-xy-yz-zx=$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

R.H.S

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

$$=\frac{(x-y)^2}{(\sqrt{2})^2}+\frac{(y-z)^2}{(\sqrt{2})^2}+\frac{(z-x)^2}{(\sqrt{2})^2}$$

$$=\frac{x^2+y^2-2xy}{2}+\frac{y^2+z^2-2yz}{2}+\frac{z^2+x^2-2zx}{2}$$

$$=\frac{x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx}{2}$$

$$=\frac{2x^2+2y^2+2z^2-2xy-2yz-2zx}{2}$$

$$=\frac{2(x^2+y^2+z^2-xy-yz-zx)}{2}$$

$$=x^2+y^2+z^2-xy-yz-zx$$

$$=\mathrm{L.H.S}$$

Q #5 **Write $2[x^2+y^2+y^2-xy-yz-zx]$ as the sum of three squares.**

Solution:

$$2[x^2+y^2+y^2-xy-yz-zx]$$

$$2x^2+2y^2+2z^2-2xy-2yz-2zx$$

$$x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx$$

Re-arranging the terms

$$x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx$$

As we have

$$\boldsymbol{a^2+b^2-2ab=(a-b)^2}$$

$$(x-y)^2+(y-z)^2+(z-x)^2$$

Ex # 4.4

Q #6 **Find the value of**
$a^2+b^2+c^2-ab-bc-ca$
when $a-b=2,\ b-c=3, c-a=4.$

Solution:

Given that:

$$a-b=2,\qquad b-c=3, c-a=4$$

To find

$$a^2+b^2+c^2-ab-bc-ca=?$$

As we have

$$\boldsymbol{2(a^2+b^2+c^2-ab-bc-ca)=(a-b)^2+(b-c)^2+(c-a)^2}$$

Put the values

$$2(a^2+b^2+c^2-ab-bc-ca)=(2)^2+(3)^2+(4)^2$$

$$2(a^2+b^2+c^2-ab-bc-ca)=4+9+16$$

$$2(a^2+b^2+c^2-ab-bc-ca)=29$$

Divide B.S by 2

$$\frac{2(a^2+b^2+c^2-ab-bc-ca)}{2}=\frac{29}{2}$$

$$a^2+b^2+c^2-ab-bc-ca=\frac{29}{2}$$

Ex # 4.5

1. $(a+b)^3=a^3+b^3+3ab(a+b)$ **Q#1, 7**
2. $(a-b)^3=a^3-b^3-3ab(a-b)$ **Q#2**
3. $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$ **Q#3**
4. $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ ***Q#4***
5. $\left(3a+\frac{1}{a}\right)^3=27a^3+\frac{1}{a^3}+3(3a)\left(\frac{1}{a}\right)\left(3a+\frac{1}{a}\right)$ **Q#5**
6. $\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3(x)\left(\frac{1}{2x}\right)\left(x-\frac{1}{2x}\right)$ **Q#6**
7. $(u-v)^3=u^3-v^3-3uv(u-v)$ **Q#8**
8. $\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2(a)\left(\frac{1}{a}\right)$ **Q#9**
9. $\left(a^2+\frac{1}{a^2}\right)^2=a^4+\frac{1}{a^4}+2(a^2)\left(\frac{1}{a^2}\right)$ **Q#9**
10. $\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

Ex # 4.5

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Q1: Find the value of $a^3 + b^3$, when

(i) $a + b = 4$ and $ab = 5$.

Solution:

$$a + b = 4, \quad ab = 5$$

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

Put the values

$(4)^3 = a^3 + b^3 + 3(5)(4)$

$64 = a^3 + b^3 + 60$

Subtract 60 from B.S

$64 - 60 = a^3 + b^3 + 60 - 60$

$4 = a^3 + b^3$

$a^3 + b^3 = 4$

(ii) $a + b = 3$ and $ab = 20$.

Solution:

$a + b = 3$ and $ab = 20$.

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

Put the values

$(3)^3 = a^3 + b^3 + 3(3)(20)$

$27 = a^3 + b^3 + 180$

Subtract 180 from B.S

$27 - 180 = a^3 + b^3 + 180 - 180$

$-153 = a^3 + b^3$

$a^3 + b^3 = -153$

(iii) $a + b = 4$ and $ab = 2$.

Solution:

$a + b = 4$ and $ab = 2$.

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

Put the values

$(4)^3 = a^3 + b^3 + 3(2)(4)$

$64 = a^3 + b^3 + 24$

Ex # 4.5

Subtract 24 from B.S

$64 - 24 = a^3 + b^3 + 24 - 24$

$40 = a^3 + b^3$

$a^3 + b^3 = 40$

Q2: Find the value of $a^3 - b^3$, when

(i) $a - b = 5$ and $ab = 7$.

Solution:

$a - b = 5, \; ab = 7$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Put the values

$(5)^3 = a^3 - b^3 - 3(7)(5)$

$125 = a^3 - b^3 - 105$

Add 105 on B.S

$125 + 105 = a^3 - b^3 - 105 + 105$

$230 = a^3 - b^3$

$a^3 - b^3 = 230$

(ii) $a - b = 2$ and $ab = 15$.

Solution:

$a - b = 2, \quad ab = 15$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Put the values

$(2)^3 = a^3 - b^3 - 3(15)(2)$

$8 = a^3 - b^3 - 90$

Add 90 on B.S

$8 + 90 = a^3 - b^3 - 90 + 90$

$98 = a^3 - b^3$

$a^3 - b^3 = 98$

Chapter # 4

Ex # 4.5

(iii) $\boldsymbol{a-b=7}$ **and** $\boldsymbol{ab=6}$**.**

Solution:

$a-b=7,\quad ab=6$

To Find:

$$a^3-b^3=?$$

As we have

$$\boldsymbol{(a-b)^3=a^3-b^3-3ab(a-b)}$$

Put the values

$(7)^3=a^3-b^3-3(6)(7)$

$343=a^3-b^3-126$

Add 126 on B.S

$343+126=a^3-b^3-126+126$

$469=a^3-b^3$

$a^3+b^3=469$

Q3: **Find the value of** $\boldsymbol{x^3+\frac{1}{x^3}}$ **, when**

(i) $\boldsymbol{x+\frac{1}{x}=\frac{5}{2}}$

Solution:

$x+\frac{1}{x}=\frac{5}{2}$

To Find:

$x^3+\frac{1}{x^3}=?$

As we have

$$\boldsymbol{\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)}$$

Put the values

$\left(\frac{5}{2}\right)^3=x^3+\frac{1}{x^3}+3\left(\frac{5}{2}\right)$

$\frac{125}{8}=x^3+\frac{1}{x^3}+\frac{15}{2}$

Subtract $\frac{15}{2}$ from B.S

$\frac{125}{8}-\frac{15}{2}=x^3+\frac{1}{x^3}+\frac{15}{2}-\frac{15}{2}$

$\frac{125-60}{8}=x^3+\frac{1}{x^3}$

$\frac{65}{8}=x^3+\frac{1}{x^3}$

$x^3+\frac{1}{x^3}=\frac{65}{8}$

Ex # 4.5

(ii) $\boldsymbol{x+\frac{1}{x}=2}$

Solution:

$$x+\frac{1}{x}=2$$

To Find:

$$x^3+\frac{1}{x^3}=?$$

As we have

$$\boldsymbol{\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)}$$

Put the values

$(2)^3=x^3+\frac{1}{x^3}+3(2)$

$8=x^3+\frac{1}{x^3}+6$

Subtract 6 from B.S

$8-6=x^3+\frac{1}{x^3}+6-6$

$2=x^3+\frac{1}{x^3}$

$x^3+\frac{1}{x^3}=2$

Q3: **Find the value of** $\boldsymbol{x^3-\frac{1}{x^3}}$ **, when**

(i) $\boldsymbol{x-\frac{1}{x}=\frac{3}{2}}$

Solution:

$$x-\frac{1}{x}=\frac{3}{2}$$

To Find:

$$x^3-\frac{1}{x^3}=?$$

As we have

$$\boldsymbol{\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)}$$

Put the values

$\left(\frac{3}{2}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{3}{2}\right)$

$\frac{27}{8}=x^3-\frac{1}{x^3}-\frac{9}{2}$

Add $\frac{9}{2}$ on B.S

$\frac{27}{8}+\frac{9}{2}=x^3-\frac{1}{x^3}-\frac{9}{2}+\frac{9}{2}$

Chapter # 4

Ex # 4.5

$$\frac{27+36}{8} = x^3 - \frac{1}{x^3}$$

$$\frac{63}{8} = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = \frac{63}{8}$$

(ii) $\boldsymbol{x - \frac{1}{x} = \frac{7}{3}}$

Solution:

$$x - \frac{1}{x} = \frac{7}{3}$$

To Find:

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

Put the values

$$\left(\frac{7}{3}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{7}{3}\right)$$

$$\frac{343}{27} = x^3 - \frac{1}{x^3} - \frac{21}{3}$$

Add $\frac{21}{3}$ on B.S

$$\frac{343}{27} + \frac{21}{3} = x^3 - \frac{1}{x^3} - \frac{21}{3} + \frac{21}{3}$$

$$\frac{343+189}{27} = x^3 - \frac{1}{x^3}$$

$$\frac{532}{27} = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = \frac{532}{27}$$

(iii) $\boldsymbol{x - \frac{1}{x} = \frac{15}{4}}$

Solution:

$$x - \frac{1}{x} = \frac{15}{4}$$

To Find:

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

Ex # 4.5

Put the values

$$\left(\frac{15}{4}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{15}{4}\right)$$

$$\frac{3375}{64} = x^3 - \frac{1}{x^3} - \frac{45}{4}$$

Add $\frac{45}{4}$ on B.S

$$\frac{3375}{64} + \frac{45}{4} = x^3 - \frac{1}{x^3} - \frac{45}{4} + \frac{45}{4}$$

$$\frac{3375+720}{64} = x^3 - \frac{1}{x^3}$$

$$\frac{4095}{64} = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = \frac{4095}{64}$$

Q5: **If $\boldsymbol{3a + \frac{1}{a} = 4}$, find $\boldsymbol{27a^3 + \frac{1}{a^3}}$**

Solution:

$$3a + \frac{1}{a} = 4$$

To Find:

$$27a^3 + \frac{1}{a^3} = ?$$

As we have

$$\left(3a + \frac{1}{a}\right)^3 = 27a^3 + \frac{1}{a^3} + 3(3a)\left(\frac{1}{a}\right)\left(3a + \frac{1}{a}\right)$$

Put the values

$$(4)^3 = 27a^3 + \frac{1}{a^3} + 9(4)$$

$$64 = 27a^3 + \frac{1}{a^3} + 36$$

Subtract 36 from B.S

$$64 - 36 = 27a^3 + \frac{1}{a^3} + 36 - 36$$

$$28 = 27a^3 + \frac{1}{a^3}$$

$$27a^3 + \frac{1}{a^3} = 28$$

Chapter # 4

Ex # 4.5

Q6: **If $x - \frac{1}{2x} = 6$, find $x^3 - \frac{1}{8x^3}$**

Solution:

$x - \frac{1}{2x} = 6$

To Find:

$x^3 - \frac{1}{8x^3} = ?$

As we have

$\left(x - \frac{1}{2x}\right)^3 = x^3 - \frac{1}{8x^3} - 3(x)\left(\frac{1}{2x}\right)\left(x - \frac{1}{2x}\right)$

Put the values

$(6)^3 = x^3 - \frac{1}{8x^3} - \frac{3}{2}(6)$

$216 = x^3 - \frac{1}{8x^3} - 3(3)$

$216 = x^3 - \frac{1}{8x^3} - 9$

Add 9 on B.S

$216 + 9 = x^3 - \frac{1}{8x^3} - 9 + 9$

$225 = x^3 - \frac{1}{8x^3}$

$x^3 - \frac{1}{8x^3} = 225$

Q7: **If $a + b = 6$, show that $a^3 + b^3 + 18ab = 216$.**

Solution:

$$a + b = 6$$

To Prove:

$$a^3 + b^3 + 18ab = 216$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$$(6)^3 = a^3 + b^3 + 3ab(6)$$
$$216 = a^3 + b^3 + 18ab$$
$$a^3 + b^3 + 18ab = 216$$

Q8: **If $u - \text{v} = 3$ then prove that $u^3 - \text{v}^3 - 9uv = 27$.**

Solution:

$$u - \text{v} = 3$$

To Prove:

$$u^3 - \text{v}^3 - 9uv = 27$$

As we have

$$(u - v)^3 = u^3 - v^3 - 3uv(u - v)$$

Ex # 4.5

Put the values

$(2)^3 = a^3 - b^3 - 3(15)(2)$

$8 = a^3 - b^3 - 90$

Add 90 on B.S

$8 + 90 = a^3 - b^3 - 90 + 90$

$98 = a^3 - b^3$

$a^3 - b^3 = 98$

Q9: **If $a + \frac{1}{a} = 2$, find the values of $a^2 + \frac{1}{a^2}$, $a^4 + \frac{1}{a^4}$, $a^3 + \frac{1}{a^3}$**

Solution:

Given

$$a + \frac{1}{a} = 2$$

To prove

$a^2 + \frac{1}{a^2} = ?$

$a^4 + \frac{1}{a^4} = ?$

$a^3 + \frac{1}{a^3} = ?$

As we have

$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2(a)\left(\frac{1}{a}\right)$

Put the values

$(2)^2 = a^2 + \frac{1}{a^2} + 2$

$4 = a^2 + \frac{1}{a^2} + 2$

Subtract 2 from B.S

$4 - 2 = a^2 + \frac{1}{a^2} + 2 - 2$

$2 = a^2 + \frac{1}{a^2}$

$a^2 + \frac{1}{a^2} = 2$

Now take square on B.S

$\left(a^2 + \frac{1}{a^2}\right)^2 = (2)^2$

$(a^2)^2 + \left(\frac{1}{a^2}\right)^2 + 2(a^2)\left(\frac{1}{a^2}\right) = 4$

$a^4 + \frac{1}{a^4} + 2 = 4$

Chapter # 4

Ex # 4.5

Subtract 2 from B.S

$$a^4+\frac{1}{a^4}+2-2=4-2$$

$$a^4+\frac{1}{a^4}=2$$

$\boldsymbol{Now\ a^3+\frac{1}{a^3}}$

Also we have

$$\boldsymbol{\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)}$$

Put the values

$$(2)^3=a^3+\frac{1}{a^3}+3(2)$$

$$8=a^3+\frac{1}{a^3}+6$$

Subtract 6 from B.S

$$8-6=a^3+\frac{1}{a^3}+6-6$$

$$2=a^3+\frac{1}{a^3}$$

$$a^3+\frac{1}{a^3}=2$$

Hence

$$a^2+\frac{1}{a^2}=a^4+\frac{1}{a^4}=a^3+\frac{1}{a^3}=2$$

Ex # 4.6

1. $a^3+b^3=(a+b)(a^2-ab+b^2)$
2. $a^3-b^3=(a-b)(a^2+ab+b^2)$
3. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)$
4. $x^3-\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)$

OR

1. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$
2. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$
3. $(x+y)(x^2-xy+y^2)=x^3+y^3$
4. $(x-y)(x^2+xy+y^2)=x^3-y^3$
5. $(x+y)(x-y)=x^2-y^2$

Ex # 4.6

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Q1: Find the following product.

(i) $\boldsymbol{(a-1)(a^2+a+1)}$

Solution:

$$(a-1)(a^2+a+1)$$
$$=(a-1)[(a)^2+(a)(1)+(1)^2]$$

As we know that

$\boldsymbol{(a-b)(a^2+ab+b^2)=a^3-b^3}$

Here $a=a$ and $b=1$

So

$=(a)^3-(1)^3$

$=a^3-1$

(ii) $\boldsymbol{(3-b)(9+3b+b^2)}$

Solution:

$$(3-b)(9+3b+b^2)$$
$$=(3-b)[(3)^2+(3)(b)+(b)^2]$$

As we know that

$\boldsymbol{(a-b)(a^2+ab+b^2)=a^3-b^3}$

Here $a=3$ and $b=b$

So

$=(3)^3-(b)^3$

$=27-b^3$

(iii) $\boldsymbol{(8+b)(64-8b+b^2)}$

Solution:

$$(8+b)(64-8b+b^2)$$
$$=(8+b)[(8)^2-(8)(b)+(b)^2]$$

As we know that

$\boldsymbol{(a+b)(a^2-ab+b^2)=a^3+b^3}$

Here $a=8$ and $b=b$

So

$=(8)^3+(b)^3$

$=512+b^3$

(iv) $(a+2)(a^2-2a+4)$

Solution:

$$(a+2)(a^2-2a+4)$$
$$=(a+2)[(a)^2-(a)(2)+(2)^2]$$

As we know that

$\boldsymbol{(a+b)(a^2-ab+b^2)=a^3+b^3}$

Here $a=a$ and $b=2$

So

$=(a)^3+(2)^3$

$=a^3+8$

Chapter # 4

Ex # 4.6

Q2: Find the following product.

(i) $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

Solution:

$\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

$\left(2p+\frac{1}{2p}\right)\left[(2p)^2+\frac{1}{(2p)^2}-(2p)\left(\frac{1}{2p}\right)\right]$

As we know that

$$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)=x^3+\frac{1}{x^3}$$

So

$=(2p)^3+\left(\frac{1}{2p}\right)^3$

$=8p^3+\frac{1}{8p^3}$

(ii) $\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

Solution:

$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left[\left(\frac{3}{2}p\right)^2+\left(\frac{2}{3p}\right)^2+\left(\frac{3}{2}p\right)\left(\frac{2}{3p}\right)\right]$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

So

$=\left(\frac{3}{2}p\right)^3-\left(\frac{2}{3p}\right)^3$

$=\frac{27}{8}p^3-\frac{8}{27p^3}$

(iii) $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

Solution:

$\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

$\left(3p-\frac{1}{3p}\right)\left[(3p)^2+\frac{1}{(3p)^2}+(3p)\left(\frac{1}{3p}\right)\right]$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

Ex # 4.6

So

$=(3p)^3-\left(\frac{1}{3p}\right)^3$

$=27p^3+\frac{1}{27p^3}$

(iv) $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

Solution:

$\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

$\left(5p+\frac{1}{5p}\right)\left[(5p)^2+\frac{1}{(5p)^2}-(5p)\left(\frac{1}{5p}\right)\right]$

As we know that

$$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)=x^3+\frac{1}{x^3}$$

So

$=(5p)^3+\left(\frac{1}{5p}\right)^3$

$=125p^3+\frac{1}{125p^3}$

Q3: Find the following continued product.

(i) $(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

Solution:

$(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

Using $a^2-b^2=(a+b)(a-b)$

$=(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

Arrange it

$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

By Using Formulas

$=(x^3+y^3)(x^3-y^3)$

Again by Formula

$=(x^3)^2-(y^3)^2$

$=x^6-y^6$

Chapter # 4

Ex # 4.6

(ii) $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

Solution:

$(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(x^2-y^2)(x^2+y^2)(x^4+y^4)$

Again by Formula

$[(x^2)^2-(y^2)^2](x^4+y^4)$

$(x^4-y^4)(x^4+y^4)$

Now again by Formula

$(x^4)^2-(y^4)^2$

x^8-y^8

iii. $(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$

Solution:

$(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$

Arrange it

$(2x-y)(4x^2+2xy+y^2)(2x+y)(4x^2-2xy+y^2)$

$(2x-y)[(2x)^2+(2x)(y)+(y)^2](2x+y)[(2x)^2-(2x)(y)+(y)^2]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

and $(x+y)(x^2-xy+y^2)=x^3+y^3$

$[(2x)^3-(y)^3][(2x)^3+(y)^3]$

$(8x^3-y^3)(8x^3+y^3)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(8x^3)^2-(y^3)^2$

$64x^6-y^6$

iv. $(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Solution:

$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Arrange it

$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

$(x-2)[(x)^2+(x)(2)+(2)^2](x+2)[(x)^2-(x)(2)+(2)^2]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

and $(x+y)(x^2-xy+y^2)=x^3+y^3$

$[(x)^3-(2)^3][(x)^3+(2)^3]$

$(x^3-8)(x^3+8)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(x^3)^2-(8)^2$

x^6-64

Ex # 4.6

Q4: Find the product with the help of formula. $(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

Solution:

$(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

$=(\sqrt{x}-\sqrt{y})\left[(\sqrt{x})^2+(\sqrt{x})(\sqrt{y})+(\sqrt{y})^2\right]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

$=(\sqrt{x})^3-(\sqrt{y})^3$

$=\left(x^{\frac{1}{2}}\right)^3-\left(y^{\frac{1}{2}}\right)^3$

$=x^{\frac{3}{2}}-y^{\frac{3}{2}}$

Q5: Simplify with the help of formula. $(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

Solution:

$(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

$=(x^p+y^q)[(x^p)^2-(x^p)(y^q)+(y^q)^2]$

As $(x+y)(x^2-xy+y^2)=x^3+y^3$

$=(x^p)^3+(y^p)^3$

$=x^{3p}+y^{3p}$

Examples Page # 116 and 117

Chapter # 4

Ex # 4.7

SURDS

A number of the form of $\sqrt[n]{a}$ is called Surd, where a is a positive rational number.

A number will be a surd, if

i. It is irrational
ii. It is a root
iii. A root of a rational number.

Examples:

$\sqrt{3}$ and $\sqrt{5+\sqrt{3}}$

In the above examples, both are irrational numbers. First number is a root of rational number 3, whereas the second number is a root of irrational number $5+\sqrt{3}$.

Thus $\sqrt{3}$ is a surd and $\sqrt{5+\sqrt{3}}$ is not a surd.

$\sqrt[3]{8}$ is not a surd because its value is 2 which is rational.

$\sqrt{-2}$, $\sqrt{-3}$ are not surds because -2 $and -3$ are negative.

Conjugate of Surds

The conjugate of $a\sqrt{x}+b\sqrt{y}$ is $a\sqrt{x}-b\sqrt{y}$.

Similarly the conjugate of $5+\sqrt{3}$ is $5-\sqrt{3}$

Ex # 4.7

Page # 122

Q1: State which of the following are surd quantities

(i) $\sqrt[3]{81}$

As 81 is a rational number and the result is irrational.
So it is surd.

(ii) $\sqrt{1+\sqrt{5}}$

As $1+\sqrt{5}$ is irrational.
So it is not surd.

(iii) $\sqrt{\sqrt{5}}$

As $\sqrt{5}$ is irrational.
So it is not surd.

(iv) $\sqrt[4]{32}$

As 32 is a rational number and the result is irrational.
So it is surd.

Ex # 4.7

(v) π

As π is irrational.
So it is not surd.

(vi) $\sqrt{1+\pi^2}$

As $1+\pi^2$ is irrational.
So it is not surd.

Q2: Express the following as the simplest possible surds.

(i) $\sqrt{12}$

Solution:

$\sqrt{12}$

$\sqrt{2\times2\times3}$

$\sqrt{2^2\times3}$

$\sqrt{2^2}\sqrt{3}$

$2\sqrt{3}$

2	12
2	6
3	3
	1

(ii) $\sqrt{48}$

Solution:

$\sqrt{48}$

$\sqrt{2\times2\times2\times2\times3}$

$\sqrt{2^2\times2^2\times3}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3}$

$2\times2\sqrt{3}$

$4\sqrt{3}$

2	48
2	24
2	12
2	6
3	3
	1

(iii) $\sqrt{240}$

Solution:

$\sqrt{240}$

$\sqrt{2\times2\times2\times2\times3\times5}$

$\sqrt{2^2\times2^2\times3\times5}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3\times5}$

$2\times2\sqrt{15}$

$4\sqrt{15}$

2	240
2	120
2	60
2	30
3	15
5	5
	1

Chapter # 4

Ex # 4.7

Q3: Simplify the following surds.

(i) $(\mathbf{2}-\sqrt{\mathbf{3}})(\mathbf{3}+\sqrt{\mathbf{5}})$

Solution:

$(2-\sqrt{3})(3+\sqrt{5})$

$2(3+\sqrt{5})-\sqrt{3}(3+\sqrt{5})$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{3\times 5}$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{15}$

(ii) $(\sqrt{\mathbf{3}}-\mathbf{4})(\sqrt{\mathbf{2}}+\mathbf{1})$

Solution:

$(\sqrt{3}-4)(\sqrt{2}+1)$

$\sqrt{3}(\sqrt{2}+1)-4(\sqrt{2}+1)$

$\sqrt{3\times 2}+1\sqrt{3}-4\sqrt{2}-4$

$\sqrt{6}+\sqrt{3}-4\sqrt{2}-4$

(iii) $(\sqrt{\mathbf{2}}+\sqrt{\mathbf{3}})(\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}})$

Solution:

$(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{2})$

$\sqrt{2}(\sqrt{5}+\sqrt{2})+\sqrt{3}(\sqrt{5}+\sqrt{2})$

$\sqrt{2\times 5}+\sqrt{2\times 2}+\sqrt{3\times 5}+\sqrt{3\times 2}$

$\sqrt{10}+2+\sqrt{15}+\sqrt{6}$

(iv) $(\mathbf{3}-\mathbf{2}\sqrt{\mathbf{3}})(\mathbf{3}+\mathbf{2}\sqrt{\mathbf{3}})$

Solution:

$(3-2\sqrt{3})(3+2\sqrt{3})$

Using Formula: $(a+b)(a+b)=a^2-b^2$

So

$(3)^2-(2\sqrt{3})^2$

$9-(2)^2(\sqrt{3})^2$

$9-4(3)$

$9-12$

-3

Q4: Rationalize the denominator and simplify.

(i) $$\frac{\mathbf{1}}{\sqrt{\mathbf{7}}}$$

Solution:

$$\frac{1}{\sqrt{7}}$$

Ex # 4.7

Multiply and divide by $\sqrt{7}$

$\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$

$\frac{1\sqrt{7}}{(\sqrt{7})^2}$

$\frac{\sqrt{7}}{7}$

(ii) $\frac{\mathbf{3}}{\sqrt{\mathbf{45}}}$

Solution:

$\frac{3}{\sqrt{45}}$

$\frac{3}{\sqrt{3\times 3\times 5}}$

$\frac{3}{3\sqrt{5}}$

$\frac{1}{\sqrt{5}}$

Multiply and divide by $\sqrt{5}$

$\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$

$\frac{1\sqrt{5}}{(\sqrt{5})^2}$

$\frac{\sqrt{5}}{5}$

(iii) $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}-\mathbf{1}}$

Solution:

$\frac{1}{\sqrt{2}-1}$

Multiply and divide by $\sqrt{2}+1$

$\frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\frac{1(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2}$

$\frac{\sqrt{2}+1}{2-1}$

$\sqrt{2}+1$

Chapter # 4

Ex # 4.7

(iv) $\mathbf{\dfrac{5}{2+\sqrt{5}}}$

Solution:

$\dfrac{5}{2+\sqrt{5}}$

Multiply and divide by $2-\sqrt{5}$

$\dfrac{5}{2+\sqrt{5}}\times\dfrac{2-\sqrt{5}}{2-\sqrt{5}}$

$\dfrac{5(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2}$

$\dfrac{5(2-\sqrt{5})}{4-5}$

$\dfrac{5(2-\sqrt{5})}{-1}$

$-5(2-\sqrt{5})$

(v) $\mathbf{\dfrac{1}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}+2}}$

Solution:

$\dfrac{1}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}+2}$

$\dfrac{1(\sqrt{5}+2)+1(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$\dfrac{\sqrt{5}+2+\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$

$\dfrac{\sqrt{5}+\sqrt{5}}{5-4}$

$\dfrac{2\sqrt{5}}{1}$

$2\sqrt{5}$

Q5: **If $x=\sqrt{5}+2$, find the value of $x+\dfrac{1}{x}$ and $x^2+\dfrac{1}{x^2}$**

Solution:

$x=\sqrt{5}+2$

To find:

$$x+\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$\dfrac{1}{x}=\dfrac{1}{\sqrt{5}+2}$

Ex # 4.7

Multiply and divide by $\sqrt{5}-2$

$\dfrac{1}{x}=\dfrac{1}{\sqrt{5}+2}\times\dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

$\dfrac{1}{x}=\dfrac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{5-4}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{1}$

$\dfrac{1}{x}=\sqrt{5}-2$

Now

$x+\dfrac{1}{x}=(\sqrt{5}+2)+(\sqrt{5}-2)$

$x+\dfrac{1}{x}=\sqrt{5}+2+\sqrt{5}-2$

$x+\dfrac{1}{x}=2\sqrt{5}$

Taking Square on B.S

$\left(x+\dfrac{1}{x}\right)^2=(2\sqrt{5})^2$

$x^2+\dfrac{1}{x^2}+2(x)\left(\dfrac{1}{x}\right)=(2)^2(\sqrt{5})^2$

$x^2+\dfrac{1}{x^2}+2=4(5)$

$x^2+\dfrac{1}{x^2}+2=20$

Subtract 2 from B.S

$x^2+\dfrac{1}{x^2}+2-2=20-2$

$x^2+\dfrac{1}{x^2}=18$

Answers:

$$x+\frac{1}{x}=2\sqrt{5}$$

$$x^2+\frac{1}{x^2}=18$$

Chapter # 4

Ex # 4.7

Q6: **If $x = \sqrt{2} + \sqrt{3}$, find the value of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$**

Solution:

$x = \sqrt{2} + \sqrt{3}$

To find:

$$x - \frac{1}{x} = ? \text{ and } x^2 + \frac{1}{x^2} = ?$$

$\frac{1}{x} = \frac{1}{\sqrt{2} + \sqrt{3}}$

Multiply and divide by $\sqrt{2} - \sqrt{3}$

$\frac{1}{x} = \frac{1}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}}$

$\frac{1}{x} = \frac{1(\sqrt{2} - \sqrt{3})}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})}$

$\frac{1}{x} = \frac{\sqrt{2} - \sqrt{3}}{(\sqrt{2})^2 - (\sqrt{3})^2}$

$\frac{1}{x} = \frac{\sqrt{2} - \sqrt{3}}{2 - 3}$

$\frac{1}{x} = \frac{\sqrt{2} - \sqrt{3}}{-1}$

$\frac{1}{x} = -(\sqrt{2} - \sqrt{3})$

$\frac{1}{x} = -\sqrt{2} + \sqrt{3}$

Now

$x - \frac{1}{x} = (\sqrt{2} + \sqrt{3}) - (-\sqrt{2} + \sqrt{3})$

$x - \frac{1}{x} = \sqrt{2} + \sqrt{3} + \sqrt{2} - \sqrt{3}$

$x - \frac{1}{x} = 2\sqrt{2}$

Taking Square on B.S

$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{2})^2$

$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = (2)^2(\sqrt{2})^2$

$x^2 + \frac{1}{x^2} - 2 = 4(2)$

$x^2 + \frac{1}{x^2} - 2 = 8$

Ex # 4.7

Add 2 on B.S

$x^2 + \frac{1}{x^2} - 2 + 2 = 8 + 2$

$x^2 + \frac{1}{x^2} = 10$

Answers:

$$x - \frac{1}{x} = 2\sqrt{2}$$

$$x^2 + \frac{1}{x^2} = 10$$

Q7: **If $x = 5 - 2\sqrt{6}$, find the value of $x + \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$**

Solution:

$x = 5 - 2\sqrt{6}$

To find:

$$x + \frac{1}{x} = ? \text{ and } x^2 + \frac{1}{x^2} = ?$$

$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}}$

Multiply and divide by $5 + 2\sqrt{6}$

$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$

$\frac{1}{x} = \frac{1(5 + 2\sqrt{6})}{(5 - 2\sqrt{6})(5 + 2\sqrt{6})}$

$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{(5)^2 - (2\sqrt{6})^2}$

$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - (2)^2(\sqrt{6})^2}$

$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - (4)(6)}$

$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - 24}$

$\frac{1}{x} = \frac{5 + 2\sqrt{6}}{1}$

$\frac{1}{x} = 5 + 2\sqrt{6}$

Chapter # 4

Ex # 4.7

Now

$$x+\frac{1}{x}=\left(5-2\sqrt{6}\right)+\left(5+2\sqrt{6}\right)$$

$$x+\frac{1}{x}=5-2\sqrt{6}+5+2\sqrt{6}$$

$$x+\frac{1}{x}=10$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(10)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=100$$

$$x^2+\frac{1}{x^2}+2=100$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=100-2$$

$$x^2+\frac{1}{x^2}=98$$

Answers:

$$x+\frac{1}{x}=10$$

$$x^2+\frac{1}{x^2}=98$$

Q8: **If $x=\frac{1}{\sqrt{2}-1}$ find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$$x=\frac{1}{\sqrt{2}-1}$$

To find

$$x-\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

Now

$$\frac{1}{x}=\sqrt{2}-1$$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}$$

Multiply and divide by $\sqrt{5}-2$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

Ex # 4.7

$$\frac{1}{x}=\frac{1\left(\sqrt{5}-2\right)}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{x}=\frac{\sqrt{5}-2}{1}$$

$$\frac{1}{x}=\sqrt{5}-2$$

Now

$$x-\frac{1}{x}=\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)$$

$$x-\frac{1}{x}=\sqrt{2}+1-\sqrt{2}+1$$

$$x-\frac{1}{x}=2$$

Taking Square on B.S

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=(2)^2$$

$$x^2+\frac{1}{x^2}-2=4$$

Add 2 on B.S

$$x^2+\frac{1}{x^2}-2+2=4+2$$

$$x^2+\frac{1}{x^2}=6$$

Answers:

$$x-\frac{1}{x}=2$$

$$x^2+\frac{1}{x^2}=6$$

Chapter # 4

Ex # 4.7

Q9: **If $x=\sqrt{10}+3$, find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$x=\sqrt{10}+3$

To find

$$x-\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

Now

$\frac{1}{x}=\frac{1}{\sqrt{10}+3}$

Multiply and divide by $\sqrt{10}-3$

$\frac{1}{x}=\frac{1}{\sqrt{10}+3}\times\frac{\sqrt{10}-3}{\sqrt{10}-3}$

$\frac{1}{x}=\frac{1(\sqrt{10}-3)}{(\sqrt{10}+3)(\sqrt{10}-3)}$

$\frac{1}{x}=\frac{\sqrt{10}-3}{(\sqrt{10})^2-(3)^2}$

$\frac{1}{x}=\frac{\sqrt{10}-3}{10-9}$

$\frac{1}{x}=\frac{\sqrt{10}-3}{1}$

$\frac{1}{x}=\sqrt{10}-3$

Now

$x-\frac{1}{x}=(\sqrt{10}+3)-(\sqrt{10}-3)$

$x-\frac{1}{x}=\sqrt{10}+3-\sqrt{10}+3$

$x-\frac{1}{x}=\sqrt{10}-\sqrt{10}+3+3$

$x-\frac{1}{x}=6$

Taking Square on B.S

$\left(x-\frac{1}{x}\right)^2=(6)^2$

$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=36$

Ex # 4.7

$x^2+\frac{1}{x^2}-2=36$

Add 2 on B.S

$x^2+\frac{1}{x^2}-2+2=36+2$

$x^2+\frac{1}{x^2}=38$

Answers:

$$x-\frac{1}{x}=6$$

$$x^2+\frac{1}{x^2}=38$$

Q10: **If $x=2-\sqrt{3}$, find the value of $x^4+\frac{1}{x^4}$**

Solution:

$x=2-\sqrt{3}$

To find

$$x+\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

Now

$\frac{1}{x}=\frac{1}{2-\sqrt{3}}$

Multiply and divide by $2+\sqrt{3}$

$\frac{1}{x}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\frac{1}{x}=\frac{1(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

$\frac{1}{x}=\frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$\frac{1}{x}=\frac{2+\sqrt{3}}{4-3}$

$\frac{1}{x}=\frac{2+\sqrt{3}}{1}$

$\frac{1}{x}=2+\sqrt{3}$

Now

$x+\frac{1}{x}=(2-\sqrt{3})+(2+\sqrt{3})$

$x+\frac{1}{x}=2-\sqrt{3}+2+\sqrt{3}$

Ex # 4.7

$$x+\frac{1}{x}=2+2-\sqrt{3}+\sqrt{3}$$

$$x+\frac{1}{x}=4$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(4)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=16$$

$$x^2+\frac{1}{x^2}+2=16$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=16-2$$

$$x^2+\frac{1}{x^2}=14$$

Again take the square on B.S

$$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$$

$$x^4+\frac{1}{x^4}+2(x^2)\left(\frac{1}{x^2}\right)=196$$

$$x^4+\frac{1}{x^4}+2=196$$

Subtract 2 from B.S

$$x^4+\frac{1}{x^4}+2-2=196-2$$

$$x^4+\frac{1}{x^4}=194$$

Answer:

$$x^4+\frac{1}{x^4}=194$$

Review Exercise # 4

Page # 124

Q2: **Simplify** $\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$

Solution:

$$\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$$

$$\frac{3y^3}{5}\cdot\frac{3a^2}{4x^3}$$

$$\frac{9y^3a^2}{20x^3}$$

$$\frac{9a^2y^3}{20x^3}$$

Q3: **Evaluate** $\frac{2x-3}{x^2-x+1}$ **for** $x=2$

Solution:

$$\frac{2x-3}{x^2-x+1}$$

Put the value

$$\frac{2x-3}{x^2-x+1}=\frac{2(2)-3}{(2)^2-(2)+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{4-3}{4-2+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{1}{2+1}$$

$$\frac{2x-3}{x^2-x+1}=\frac{1}{3}$$

Q4: **Find the value of** x^2+y^2 **and** xy **when** $x+y=7$, $x-y=3$.

Solution:

$$x+y=7\ ,\ x-y=3$$

To Find:

x^2+y^2 =? and xy =?

$\underline{x^2+y^2}$

As we have

$$(x+y)^2+(x-y)^2=2(x^2+y^2)$$

Put the values

$$(7)^2+(3)^2=2(x^2+y^2)$$

$$49+9=2(x^2+y^2)$$

$$58=2(x^2+y^2)$$

Chapter # 4

Review Ex # 4

Divide B.S by 2

$$\frac{58}{2} = \frac{2(x^2+y^2)}{2}$$

$29 = x^2 + y^2$

$29 = x^2 + y^2$

xy

As we have

$$(\boldsymbol{x}+\boldsymbol{y})^2 - (\boldsymbol{x}-\boldsymbol{y})^2 = \boldsymbol{4xy}$$

Put the values

$(7)^2 - (3)^2 = 4xy$

$49 - 9 = 4xy$

$40 = 4xy$

Divide B.S by 4

$$\frac{40}{4} = \frac{4xy}{4}$$

$10 = xy$

$xy = 10$

Q5: Find the value of $\boldsymbol{a+b+c}$ when $\boldsymbol{a^2+b^2+c^2 = 43}$ and $\boldsymbol{ab+bc+ca = 3}$.

Solution:

$a^2 + b^2 + c^2 = 43$ and $ab + bc + ca = 3$

To Find:

$$a + b + c = ?$$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 + \boldsymbol{c}^2 + 2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Put the values

$(a+b+c)^2 = 43 + 2(3)$

$(a+b+c)^2 = 43 + 6$

$(a+b+c)^2 = 49$

Taking square root on B.S

$\sqrt{(a+b+c)^2} = \sqrt{49}$

$a + b + c = 7$

Q6: If $\boldsymbol{a+b+c = 6}$ and $\boldsymbol{a^2+b^2+c^2 = 24}$, then find the value of $\boldsymbol{ab+bc+ca}$

Solution:

$a + b + c = 6 \; and \; a^2 + b^2 + c^2 = 24$

To Find:

$$ab + bc + ca = ?$$

As we know that

$$(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 + \boldsymbol{c}^2 + 2(\boldsymbol{ab}+\boldsymbol{bc}+\boldsymbol{ca})$$

Review Ex # 4

Put the values

$(6)^2 = 24 + 2(ab + bc + ca)$

$36 = 25 + 2(ab + bc + ca)$

Subtract 24 from B.S

$$36 - 24 = 24 - 24 + 2(ab + bc + ca)$$

$12 = 2(ab + bc + ca)$

Divide B.S by 2

$$\frac{12}{2} = \frac{2(ab+bc+ca)}{2}$$

$6 = ab + bc + ca$

$ab + bc + ca = 6$

Q7: If $\boldsymbol{2x - 3y = 8}$ and $\boldsymbol{xy = 2}$, then find the values of $\boldsymbol{8x^3 - 27y^3}$.

Solution:

$2x - 3y = 8$ and $xy = 2$

To Find:

$$8x^3 - 27y^3 = ?$$

As we have

$$(\boldsymbol{2x}-\boldsymbol{3y})^3 = (\boldsymbol{2x})^3 - (\boldsymbol{3y})^3 - 3(\boldsymbol{2x})(\boldsymbol{3y})(\boldsymbol{2x}-\boldsymbol{3y})$$

Put the values

$(8)^3 = 8x^3 - 27y^3 - 18xy(8)$

$512 = 8x^3 - 27y^3 - 18(2)(8)$

$512 = 8x^3 - 27y^3 - 288$

Add 288 on B.S

$512 + 288 = 8x^3 - 27y^3 - 288 + 288$

$800 = 8x^3 - 27y^3$

$8x^3 - 27y^3 = 800$

Chapter # 4

Review Ex # 4

Q8: **Find the product** $\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2-\frac{25}{16x^2}+1\right)$

Solution:

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2-\frac{25}{16x^2}+1\right)$$

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2+\left(\frac{5}{4x}\right)^2+\left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right)\right]$$

As we know that

$$\left(\boldsymbol{x}-\frac{\mathbf{1}}{\boldsymbol{x}}\right)\left(\boldsymbol{x}^2+\frac{\mathbf{1}}{\boldsymbol{x}^2}+(\boldsymbol{x})\left(\frac{\mathbf{1}}{\boldsymbol{x}}\right)\right)=\boldsymbol{x}^3-\frac{\mathbf{1}}{\boldsymbol{x}^3}$$

$$=\left(\frac{4}{5}x\right)^3-\left(\frac{5}{4x}\right)^3$$

$$=\frac{64}{125}x^3-\frac{125}{64x^3}$$

Q9: **Find the value of** $\boldsymbol{x^3+\frac{1}{x^3}}$ **, when** $\boldsymbol{x+\frac{1}{x}=8}$

Solution:

$$x+\frac{1}{x}=8$$

To Find:

$$x^3+\frac{1}{x^3}=?$$

As we have

$$\left(\boldsymbol{x}+\frac{\mathbf{1}}{\boldsymbol{x}}\right)^3=\boldsymbol{x}^3+\frac{\mathbf{1}}{\boldsymbol{x}^3}+\mathbf{3}(\boldsymbol{x})\left(\frac{\mathbf{1}}{\boldsymbol{x}}\right)\left(\boldsymbol{x}+\frac{\mathbf{1}}{\boldsymbol{x}}\right)$$

Put the values

$$(8)^3=x^3+\frac{1}{x^3}+3(8)$$

$$512=x^3+\frac{1}{x^3}+24$$

Subtract 24 from B.S

$$512-24=x^3+\frac{1}{x^3}+24-24$$

$$488=x^3+\frac{1}{x^3}$$

$$x^3+\frac{1}{x^3}=488$$

Review Ex # 4

Think **Trick**

Q10: **Simplify** $\boldsymbol{\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}}$

Solution:

$$\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

$$\frac{2x^2}{x^4-16}+\frac{1}{x+2}-\frac{x}{x^2-4}$$

$$\frac{2x^2}{(x^2)^2-(4)^2}+\frac{1}{x+2}-\frac{x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{1(x-2)-x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{x-2-x}{(x+2)(x-2)}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{x-x-2}{x^2-4}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}+\frac{-2}{x^2-4}$$

$$\frac{2x^2}{(x^2+4)(x^2-4)}-\frac{2}{x^2-4}$$

$$\frac{2x^2-2(x^2+4)}{(x^2+4)(x^2-4)}$$

$$\frac{2x^2-2x^2-8}{(x^2)^2-(4)^2}$$

$$\frac{-8}{x^4-16}$$

Chapter # 5

UNIT # 5

FACTORIZATION

Ex # 5.1

Factorization

Writing an algebraic expression as the product of two or more algebraic expressions is called factorization of the algebraic expression.

Example

$$\mathbf{5x + 10x^2} = 5x(1 + 2x)$$

Here $5x$ and $1 + 2x$ are called factors of $5x + 10x^2$.

Type 1: $ka + kb + kc$

Common Techniques

اگر کسی سوال سے پہلے minus آجائیں تو اس کو common لیں گے۔ minus کو common لینے سے تمام terms کے sign تبدیل ہو جائیں گے۔

$\mathbf{-2x^3 + 12y - 7z} = -(2x^3 - 12y + 7z)$

اگر تمام terms کے constants ایک ہی table میں آتے ہو تو اس میں بھی common لیں گے۔

$\mathbf{4x^3 - 24y + 64} = 4(x^3 - 6y + 16)$

اگر تمام terms میں ایک جیسے variable ہو تو سب سے کم power والے variable کو common لیں گے۔

$\mathbf{4x^3 - 5x^2 + 3xy} = x(4x^2 - 5x + 3y)$

Example:

$\mathbf{-4x^3 + 24x^2 - 64x} = -4x(x^2 - 6x + 16)$

Example 1:

(i) $\mathbf{15 + 10x - 5x^2} = 5(1 + 2x - x^2)$

(ii) $\mathbf{12x^2y^2 - 20x^3y} = 4x^2y(3y - 5x)$

Type 2: $ac + ad + bc + bd$.

Example 2:

Factorize $a^2 - ab - 3a + 3b$

Solution:

$a^2 - ab - 3a + 3b$ **Making two pairs/groups**

Taking common from each group

$= a(a - b) - 3(a - b)$

$= (a - b)(a - 3)$ As $(a - b)$ is a common

Ex # 5.1

Type 3: $a^2 \pm 2ab + b^2$

Example 3:

$\mathbf{x^2 + 8x + 16} = (x)^2 + 2(x)(4) + (4)^2$

$x^2 + 8x + 16 = (x + 4)^2 = (x + 4)(x + 4)$

$\mathbf{25y^2 - 30y + 9} = (5y)^2 - 2(5y)(3) + (3)^2$

$25y^2 - 30y + 9 = (5y - 3)^2 = (5y - 3)(5y - 3)$

Type 4: $a^2 - b^2$

Example 4:

(i) $\mathbf{x^2 - 16} = (x)^2 - (4)^2 = (x + 4)(x - 4)$

(ii) $\mathbf{9a^2 - 25} = (3a)^2 - (5)^2 = (3a + 5)(3a - 5)$

(iii) $\mathbf{6x^4 - 6y^4} = 6(x^4 - y^4)$

$6x^4 - 6y^4 = 6[(x^2)^2 - (y^2)^2]$

$6x^4 - 6y^4 = 6(x^2 + y^2)(x^2 - y^2)$

$6x^4 - 6y^4 = 6(x^2 + y^2)(x + y)(x - y)$

Type 5: $a^2 \pm 2ab + b^2 - c^2$

Example 5:

Factorize $a^2 + 4ab + 4b^2 - c^2$

Solution:

$a^2 + 4ab + 4b^2 - c^2$

$= (a)^2 + 2(a)(2b) + (2b)^2 - c^2$

$= (a + 2b)^2 - (c)^2$

$= (a + 2b + c)(a + 2b - c)$

Example 6:

Factorize $a^2 - b^2 + 2b - 1$

Solution:

$a^2 - b^2 + 2b - 1$

$= a^2 - (b^2 - 2b + 1)$

$= a^2 - \{(b)^2 - 2(b)(1) + (1)^2\}$

$= a^2 - (b - 1)^2$

$= \{a + (b - 1)\}\{a - (b - 1)\}$

$(a + b - 1)(a - b + 1)$

Chapter # 5

Exercise# 5.1

Q1 $\mathbf{9s^2t + 15s^2t^3 - 3s^2t^2}$

Solution

$9s^2t + 15s^2t^3 - 3s^2t^2$

Take common $3s^2t$

$= 3s^2t(3 + 5t^2 - t)$

Q2 $\mathbf{10a^2b^3c^4 - 15a^3b^2c^2 + 30a^4b^3c^2}$

Solution

$10a^2b^3c^4 - 15a^3b^2c^2 + 30a^4b^3c^2$

Take common $5a^2b^2c^2$

$= 5a^2b^2c^2(2bc^2 - 3a + 6a^2b)$

Q3 $\mathbf{ax - a - x + 1}$

Solution

$ax - a - x + 1$

Taking common

$= a(x-1) - 1(x-1)$

Taking common

$= (x-1)(a-1)$

Q4 $\mathbf{x^2 - 2y^3 - 2xy^2 + xy}$

Solution

$x^2 - 2y^3 - 2xy^2 + xy$

Arrange it

$= x^2 + xy - 2xy^2 - 2y^3$

Taking common

$= x(x+y) - 2y^2(x+y)$

Taking common

$= (x+y)(x - 2y^2)$

Q5 $\mathbf{4x^2 + 4 + \frac{1}{x^2}}$

Solution

$4x^2 + 4 + \dfrac{1}{x^2}$

$= (2x)^2 + 2(2x)\dfrac{1}{x} + \left(\dfrac{1}{x}\right)^2$

As we know that

$= a^2 + 2ab + b^2 = (a+b)^2$

$= \left(2x + \dfrac{1}{x}\right)^2$

Q6 $\mathbf{4(x+y)^2 - 20(x+y)z + 25z^2}$

Solution

$4(x+y)^2 - 20(x+y)z + 25z^2$

$= [2(x+y)]^2 - 2[2(x+y)](5z) + (5z)^2$

As we know that $a^2 - 2ab + b^2 = (a-b)^2$

$= [2(x+y) - 5z]^2$

Q7 $\mathbf{\frac{x^4}{y^4} - \frac{y^4}{x^4}}$

Solution

$\dfrac{x^4}{y^4} - \dfrac{y^4}{x^4}$

$= \dfrac{(x^2)^2}{(y^2)^2} - \dfrac{(y^2)^2}{(x^2)^2}$

$= \left(\dfrac{x^2}{y^2}\right)^2 - \left(\dfrac{y^2}{x^2}\right)^2$

Using Formula $a^2 - b^2 = (a+b)(a-b)$

$= \left(\dfrac{x^2}{y^2} + \dfrac{y^2}{x^2}\right)\left(\dfrac{x^2}{y^2} - \dfrac{y^2}{x^2}\right)$

$= \left(\dfrac{x^2}{y^2} + \dfrac{y^2}{x^2}\right)\left[\left(\dfrac{x}{y}\right)^2 - \left(\dfrac{y}{x}\right)^2\right]$

Using Formula $a^2 - b^2 = (a+b)(a-b)$

$= \left(\dfrac{x^2}{y^2} + \dfrac{y^2}{x^2}\right)\left(\dfrac{x}{y} + \dfrac{y}{x}\right)\left(\dfrac{x}{y} - \dfrac{y}{x}\right)$

Q8 $\mathbf{2x^2 - 288}$

Solution

$2x^2 - 288$

Taking common

$= 2(x^2 - 144)$

$= 2[(x)^2 - (12)^2]$

Using Formula $a^2 - b^2 = (a+b)(a-b)$

$= 2(x+12)(x-12)$

Chapter # 5

Q9 $\mathbf{1 - u^2 + 2uv - v^2}$

Solution

$1 - (u^2 - 2uv + v^2)$

Using Formula $a^2 - 2ab + b^2 = (a - b)^2$

$= (1)^2 - (u - v)^2$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$= [1 + (u - v)][1 - (u - v)]$

$= (1 + u - v)(1 - u + v)$

Q10 $\mathbf{25a^2b^2 - 20abc + 4c^2 - 16d^2}$

Solution

$25a^2b^2 - 20abc + 4c^2 - 16d^2$

Using Formula $a^2 - 2ab + b^2 = (a - b)^2$

$= (5ab)^2 - 2(5ab)(2c) + (2c)^2 - (4d)^2$

$= (5ab - 2c)^2 - (4d)^2$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$= (5ab - 2c + 4d)(5ab - 2c - 4d)$

Exercise# 5.2

Q1 $\mathbf{x^4 + 64}$

Solution

$x^4 + 64$

$= (x^2)^2 + (8)^2$

Add and Subtract $2(x^2)(8)$

$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (x^2 + 8)^2 - 16x^2$

$= (x^2 + 8)^2 - (4x)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$

$= (x^2 + 4x + 8)\ (x^2 - 4x + 8)$

Q2 $\mathbf{4x^4 + 81}$

Solution

$4x^4 + 81$

$= (2x^2)^2 + (9)^2$

Add and Subtract $2(2x^2)(9)$

$= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9)$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (2x^2 + 9)^2 - 36x^2$

$= (2x^2 + 9)^2 - (6x)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$

$= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$

Q3 $a^4 + a^2b^2 + b^4$

Solution

$a^4 + a^2b^2 + b^4$

$= a^4 + b^4 + a^2b^2$

$= (a^2)^2 + (b^2)^2 + a^2b^2$

Add and Subtract $2(a^2)(b^2)$

$= (a^2)^2 + (b^2)^2 + 2(a^2)(b^2) - 2(a^2)(b^2) + a^2b^2$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$

$= (a^2 + b^2)^2 - a^2b^2$

$= (a^2 + b^2)^2 - (ab)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

Q4 $\mathbf{x^4 + x^2 + 1}$

Solution

$x^4 + x^2 + 1$

$= x^4 + 1 + x^2$

$= (x^2)^2 + (1)^2 + x^2$

Add and Subtract $2(x^2)(1)$

$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$= (x^2 + 1)^2 - 2x^2 + x^2$

$= (x^2 + 1)^2 - x^2$

As $a^2 - b^2 = (a + b)(a - b)$

$= (x^2 + 1 + x)(x^2 + 1 - x)$

$= (x^2 + x + 1)(x^2 - x + 1)$

Q5 $\mathbf{x^8 + x^4 + 1}$

Solution

$x^8 + x^4 + 1$

$= x^8 + 1 + x^4$

$= (x^4)^2 + (1)^2 + x^4$

Add and Subtract $2(x^4)(1)$

$(x^4)^2 + (1)^2 + 2(x^4)(1) - 2(x^4)(1) + x^4$

Using Formula $a^2 + b^2 + 2ab = (a + b)^2$

$(x^4 + 1)^2 - 2x^4 + x^4$

$(x^4 + 1)^2 - x^4$

$(x^4 + 1)^2 - (x^2)^2$

As $a^2 - b^2 = (a + b)(a - b)$

$(x^4 + 1 + x^2)(x^4 + 1 - x^2)$

$[(x^2)^2 + (1)^2 + x^2](x^4 + 1 - x^2)$

$[(x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2](x^4 + 1 - x^2)$

Chapter # 5

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= [(x^2+1)^2 - 2x^2 + x^2](x^4 + 1 - x^2)$

$= [(x^2+1)^2 - x^2](x^4 + 1 - x^2)$

As $a^2 - b^2 = (a+b)(a-b)$

$= [(x^2+1+x)(x^2+1-x)](x^4+1-x^2)$

$= [(x^2+x+1)(x^2-x+1)](x^4-x^2+1)$

Q6 $\boldsymbol{x^4 + \frac{1}{x^4} - 7}$

Solution

$x^4 + \dfrac{1}{x^4} - 7$

$= (x^2)^2 + \left(\dfrac{1}{x^2}\right)^2 - 7$

Add and Subtract $2(x^2)\left(\frac{1}{x^2}\right)$

$= (x^2)^2 + \left(\dfrac{1}{x^2}\right)^2 + 2(x^2)\left(\dfrac{1}{x^2}\right) - 2(x^2)\left(\dfrac{1}{x^2}\right) - 7$

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= \left(x^2 + \dfrac{1}{x^2}\right)^2 - 2 - 7$

$= \left(x^2 + \dfrac{1}{x^2}\right)^2 - 9$

$= \left(x^2 + \dfrac{1}{x^2}\right)^2 - (3)^2$

As $a^2 - b^2 = (a+b)(a-b)$

$= \left(x^2 + \dfrac{1}{x^2} + 3\right)\left(x^2 + \dfrac{1}{x^2} - 3\right)$

Q7 $\boldsymbol{81x^4 + \frac{1}{81x^4} - 14}$

Solution

$81x^4 + \dfrac{1}{81x^4} - 14$

$= (9x^2)^2 + \left(\dfrac{1}{9x^2}\right)^2 - 14$

Add and Subtract $2(9x^2)\left(\frac{1}{9x^2}\right)$

$= (9x^2)^2 + \left(\dfrac{1}{9x^2}\right)^2 + 2(9x^2)\left(\dfrac{1}{9x^2}\right) - 2(9x^2)\left(\dfrac{1}{9x^2}\right) - 14$

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= \left(9x^2 + \dfrac{1}{9x^2}\right)^2 - 2 - 14$

$= \left(9x^2 + \frac{1}{9x^2}\right)^2 - 16$

$= \left(9x^2 + \dfrac{1}{9x^2}\right)^2 - (4)^2$

As $a^2 - b^2 = (a+b)(a-b)$

$= \left(9x^2 + \dfrac{1}{9x^2} + 4\right)\left(9x^2 + \dfrac{1}{9x^2} - 4\right)$

Q8 $\boldsymbol{4x^4 - 4x^2y^2 + 64y^4}$

Solution

$4x^4 - 4x^2y^2 + 64y^4$

$= 4(x^4 - x^2y^2 + 16y^2)$

$= 4(x^4 + 16y^4 - x^2y^2)$

$= 4[(x^2)^2 + (4y^2)^2 - x^2y^2]$

Add and Subtract $2(x^2)(4y^2)$

$= 4[(x^2)^2 + (4y^2)^2 + 2(x^2)(4y^2) - 2(x^2)(4y^2) - x^2y^2]$

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= 4[(x^2+4y^2)^2 - 8x^2y^2 - x^2y^2]$

$= 4[(x^2+4y^2)^2 - 9x^2y^2]$

$= 4[(x^2+4y^2)^2 - (3xy)^2]$

As $a^2 - b^2 = (a+b)(a-b)$

$= 4(x^2+4y^2+3xy)(x^2+4y^2-3xy)$

$= 4(x^2+3xy+4y^2)(x^2-3xy+4y^2)$

Q9 $\boldsymbol{16m^4 + 4m^2n^2 + n^4}$

Solution

$16m^4 + 4m^2n^2 + n^4$

$= 16m^4 + n^4 + 4m^2n^2$

$= (4m^2)^2 + (n^2)^2 + 4m^2n^2$

Add and Subtract $2(4m^2)(n^2)$

$= (4m^2)^2 + (n^2)^2 + 2(4m^2)(n^2) - 2(4m^2)(n^2) + 4m^2n^2$

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= (4m^2+n^2)^2 - 8m^2n^2 + 4m^2n^2$

$= (4m^2+n^2)^2 - 4m^2n^2$

$= (4m^2+n^2)^2 - (2mn)^2$

As $a^2 - b^2 = (a+b)(a-b)$

$= (4m^2+n^2+2mn)(4m^2+n^2-2mn)$

$= (4m^2+2mn+n^2)(4m^2-2mn+n^2)$

Chapter # 5

Q10 $\mathbf{4x^5y + 11x^3y^3 + 9xy^5}$

Solution

$4x^5y + 11x^3y^3 + 9xy^5$

$= xy(4x^4 + 11x^2y^2 + 9y^4)$

$= xy(4x^4 + 9y^4 + 11x^2y^2)$

$= xy[(2x^2)^2 + (3y^2)^2 + 11x^2y^2]$

Add and Subtract $2(2x^2)(3y^2)$

$= xy[(2x^2)^2 + (3y^2)^2 + 2(2x^2)(3y^2) - 2(2x^2)(3y^2) + 11x^2y^2]$

Using Formula $a^2 + b^2 + 2ab = (a+b)^2$

$= xy[(2x^2 + 3y^2)^2 - 12x^2y^2 + 11x^2y^2]$

$= xy[(2x^2 + 3y^2)^2 - x^2y^2]$

$= xy[(2x^2 + 3y^2)^2 - (xy)^2]$

As $a^2 - b^2 = (a+b)(a-b)$

$= xy(2x^2 + 3y^2 + xy)(2x^2 + 3y^2 - xy)$

$= xy(2x^2 + xy + 3y^2)(2x^2 - xy + 3y^2)$

Exercise# 5.3

Q1 $\mathbf{x^2 - 7x + 12}$

Solution

$x^2 - 7x + 12$

$= x^2 - 3x - 4x + 12$

$= x(x-3) - 4(x-3)$

$= (x-3)(x-4)$

$(x^2)(12) = 12x^2$	
Add	**Multiply**
$-3x$	$-3x$
$-4x$	$-4x$
$-7x$	$12x^2$

Q2 $\mathbf{x^2 + x - 12}$

Solution

$x^2 + x - 12$

$= x^2 - 3x + 4x - 12$

$= x(x-3) + 4(x-3)$

$= (x-3)(x+4)$

$(x^2)(-12) = -12x^2$	
Add	**Multiply**
$-3x$	$-3x$
$+4x$	$+4x$
x	$-12x^2$

Q3 $\mathbf{20 - x - x^2}$

Solution

$20 - x - x^2$

$= 20 + 4x - 5x - x^2$

$= 4(5+x) - x(5+x)$

$= (5+x)(4-x)$

$(20)(-x^2) = -20x^2$	
Add	**Multiply**
$+4x$	$+4x$
$-5x$	$-5x$
$-x$	$-20x^2$

Q4 $\mathbf{2y^2 - 7y + 3}$

Solution

$= 2y^2 - 1y - 6y + 3$

$= y(2y-1) - 3(2y-1)$

$= (2y-1)(y-3)$

$(2y^2)(3) = 6y^2$	
Add	**Multiply**
$-1y$	$-1y$
$-6y$	$-6y$
$-7y$	$6y^2$

Q5 $\mathbf{4x^2 + 8x + 3}$

Solution

$4x^2 + 8x + 3$

$= 4x^2 + 2x + 6x + 3$

$= 2x(2x+1) + 3(2x+1)$

$= (2x+1)(2x+3)$

$(4x^2)(3) = 12x^2$	
Add	**Multiply**
$+2x$	$+2x$
$+6x$	$+6x$
$8x$	$12x^2$

Q6 $\mathbf{10y^2 - 3y - 1}$

Solution

$10y^2 - 3y - 1$

$= 10y^2 + 2y - 5y - 1$

$= 2y(5y+1) - 1(5y+1)$

$= (5y+1)(2y-1)$

$(10y^2)(-1) = -10y^2$	
Add	**Multiply**
$+2y$	$+2y$
$-5y$	$-5y$
$-3y$	$-10y^2$

Q7 $6x^3 - 15x^2 - 9x$

Solution

$= 3x(2x^2 - 5x - 3)$

$= 3x(2x^2 + 1x - 6x - 3)$

$= 3x[x(2x+1) - 3(2x+1)]$

$= 3x(2x+1)(x-3)$

$(2x^2)(-3) = -6x^2$	
Add	**Multiply**
$+1x$	$+1x$
$-6x$	$-6x$
$-5x$	$-6x^2$

Q8 $2xy^2 + 8xy - 24x$

Solution

$= 2x(y^2 + 4y - 12)$

$= 2x(y^2 - 2y + 6y - 12)$

$= 2x[y(y-2) + 6(y-2)]$

$= 2x(y-2)(y+6)$

$(y^2)(-12) = -12y^2$	
Add	**Multiply**
$-2y$	$-2y$
$+6y$	$+6y$
$+4y$	$-12y^2$

Q10 $-16x^3y - 20x^2y^2 - 6xy^3$

Solution

$-16x^3y - 20x^2y^2 - 6xy^3$

$= -2xy(8x^2 + 10xy + 3y^2)$

$-2xy(8x^2 + 4xy + 6xy + 3y^2)$

$-2xy[4x(2x+y) + 3y(2x+y)]$

$-2xy(2x+y)(4x+3y)$

$(8x^2)(3y^2) = 24x^2y^2$	
Add	**Multiply**
$+4xy$	$+4xy$
$+6xy$	$+6xy$
$+10xy$	$24x^2y^2$

Q11 $\mathbf{(x+1)^2 + 3(x+1) + 2}$

Solution

$(x+1)^2 + 3(x+1) + 2$

$= x^2 + (1)^2 + 2(x)(1) + 3x + 3 + 2$

$= x^2 + 1 + 2x + 3x + 5$

$= x^2 + 5x + 6$

$= x^2 + 2x + 3x + 6$

$= x(x+2) + 3(x+2)$

$= (x+2)(x+3)$

$(x^2)(6) = 6x^2$	
Add	**Multiply**
$+2x$	$+2x$
$+3x$	$+3x$
$5x$	$6x^2$

Chapter # 5

Q12 $\mathbf{4x^8y^{10} - 40x^5y^7 + 84x^2y^4}$

Solution

$(x^6y^6)(21) = 21x^6y^6$	
Add	Multiply
$-3x^3y^3$	$-3x^3y^3$
$-7x^3y^3$	$-7x^3y^3$
$-10x^3y^3$	$21x^6y^6$

$4x^8y^{10} - 40x^5y^7 + 84x^2y^4$
$= 4x^2y^4(x^6y^6 - 10x^3y^3 + 21)$
$= 4x^2y^4(x^6y^6 - 3x^3y^3 - 7x^3y^3 + 21)$
$= 4x^2y^4[x^3y^3(x^3y^3 - 3) - 7(x^3y^3 - 3)]$
$= 4x^2y^4(x^3y^3 - 3)(x^3y^3 - 7)$

Q13 **Find an expression for the perimeter of a rectangle with area given by $\mathbf{x^2 + 24x - 81}$**

Given

$Area\ of\ rectangle = x^2 + 24x - 81$

To find

Perimeter of rectangle = ?

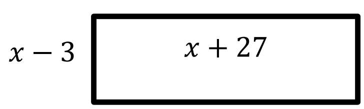
$(x^2)(-81) = -81x^2$	
Add	**Multiply**
$-3x$	$-3x$
$+27x$	$+27x$
$24x$	$-81x^2$

As $Area = l \times w$

And $Perimeter = 2l + 2w$

Now

$x^2 + 24x - 81$
$= x^2 - 3x + 27x - 81$
$= x(x - 3) + 27(x - 3)$
$= (x - 3)(x + 27)$



Now $l = (x + 27)\, and\ w = (x - 3)$

As

$Perimeter = 2l + 2w$
$Perimeter = 2(x + 27) + 2(x - 3)$
$Perimeter = 2x + 54 + 2x - 6$
$Perimeter = 4x + 48$

Q9 $\mathbf{2 + 5t - 12t^2}$

Solution

$(12t^2)(-2) = -24t^2$	
Add	**Multiply**
$+3t$	$+3t$
$-8t$	$-8t$
$-5t$	$-24t^2$

$2 + 5t - 12t^2$
$-12t^2 + 5t + 2$
$-(12t^2 - 5t - 2)$
$-(12t^2 + 3t - 8t - 2)$
$-[3t(4t + 1) - 2(4t + 1)]$
$-(4t + 1)(3t - 2)$

Exercise# 5.4

Q1 $\mathbf{(4x^2 - 16x + 7)(4x^2 - 16x + 15) + 16}$

Solution

$(4x^2 - 16x + 7)(4x^2 - 16x + 15) + 16$
Let $4x^2 - 16x = y$
$= (y + 7)(y + 15) + 16$
$= y^2 + 15y + 7y + 105 + 16$
$= y^2 + 22y + 121$
$= y^2 + 11y + 11y + 121$
$= y(y + 11) + 11(y + 11)$
$= (y + 11)(y + 11)$
But $y = 4x^2 - 16x$
$= (4x^2 - 16x + 11)\,(4x^2 - 16x + 11)$
$= (4x^2 - 16x + 11)^2$

Q2 $\mathbf{(9x^2 + 9x - 4)(9x^2 + 9x - 10) - 72}$

Solution

$(9x^2 + 9x - 4)(9x^2 + 9x - 10) - 72$
Let $9x^2 + 9x = y$
$= (y - 4)(y - 10) - 72$
$= y^2 - 10y - 4y - 40 - 72$
$= y^2 - 14y - 32$
$= y^2 + 2y - 16y - 32$
$= y(y + 2) - 16(y + 2)$
$= (y + 2)(y - 16)$
But $y = 9x^2 + 9x$
So
$= (9x^2 + 9x + 2)(9x^2 + 9x - 16)$

Q3 $\mathbf{(x + 2)(x + 4)(x + 6)(x + 8) - 9}$

Solution

$(x + 2)(x + 4)(x + 6)(x + 8) - 9$
Rearranging accordingly 4+6=2+8
$= (x + 2)(x + 8)(x + 4)(x + 6) - 9$
$= (x^2 + 8x + 2x + 16)(x^2 + 6x + 4x + 24) - 9$
$= (x^2 + 10x + 16)(x^2 + 10x + 24) - 9$
Let $x^2 + 10x = y$
$= (y + 16)(y + 24) - 9$
$= y^2 + 24y + 16y + 384 - 9$
$= y^2 + 40y + 375$
$= y^2 + 15y + 25y + 375$
$= y(y + 15) + 25(y + 15)$
$= (y + 15)(y + 25)$

Chapter # 5

But $y = x^2 + 10x$
So
$= (x^2 + 10x + 15)(x^2 + 10x + 25)$

Q4 $\boldsymbol{x(x+1)(x+2)(x+3)+1}$

Solution

$x(x+1)(x+2)(x+3)+1$
Rearranging accordingly 0+3=1+2
$= x(x+3)(x+1)(x+2)+1$
$= (x^2+3x)(x^2+2x+1x+2)+1$
$= (x^2+3x)(x^2+3x+2)+1$
Let $\boldsymbol{x^2+3x=y}$
$= (y)(y+2)+1$
$= y^2+2y+1$
$= (y)^2+(1)^2+2(y)(1)$
$= (y+1)^2$
But $\boldsymbol{y=x^2+3x}$
So
$= (x^2+3x+1)^2$

Q5 $\boldsymbol{(x+1)(x+2)(x+3)(x+6)-3x^2}$

Solution

$(x+1)(x+2)(x+3)(x+6)-3x^2$
Rearranging accordingly $1 \times 6 = 2 \times 3$
$= (x+1)(x+6)(x+2)(x+3)-3x^2$
$= (x^2+6x+1x+6)(x^2+3x+2x+6)-3x^2$
$= (x^2+7x+6)(x^2+5x+6)-3x^2$
$= (x^2+6+7x)(x^2+6+5x)-3x^2$
Let $\boldsymbol{x^2+6=y}$
$= (y+7x)(y+5x)-3x^2$
$= y^2+5xy+7xy+35x^2-3x^2$
$= y^2+12xy+32x^2$
$= y^2+4xy+8xy+32x^2$
$= y(y+4x)+8x(y+4x)$
$= (y+4x)(y+8x)$
But $\boldsymbol{y=x^2+6}$
$= (x^2+6+4x)(x^2+6+8x)$
$$= \left(\frac{x(x^2+6+4x)}{x}\right)\left(\frac{x(x^2+6+8x)}{x}\right)$$
$$= x.x\left(\frac{x^2}{x}+\frac{6}{x}+\frac{4x}{x}\right)\left(\frac{x^2}{x}+\frac{6}{x}+\frac{8x}{x}\right)$$
$$= x^2\left(x+\frac{6}{x}+4\right)\left(x+\frac{6}{x}+8\right)$$

Q6 $\boldsymbol{64x^3-144x^2y+108xy^2-27y^3}$

Solution

$64x^3-144x^2y+108xy^2-27y^3$
$= (4x)^3-3(4x)^2(3y)+3(4x)(3y)^2-(3y)^3$
As $a^3-3a^2b+3ab^2-b^3=(a-b)^3$
$= (4x-3y)^3$

Q7 $\boldsymbol{\frac{a^3}{8}-\frac{1}{4}a^2b+\frac{1}{6}ab^2-\frac{b^3}{27}}$

Solution

$$\frac{a^3}{8}-\frac{1}{4}a^2b+\frac{1}{6}ab^2-\frac{b^3}{27}$$
$$= \left(\frac{a}{2}\right)^3-3\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)+3\left(\frac{a}{2}\right)\left(\frac{b}{3}\right)^2-\left(\frac{b}{3}\right)^3$$
As $a^3-3a^2b+3ab^2-b^3=(a-b)^3$
$$= \left(\frac{a}{2}-\frac{b}{3}\right)^3$$

Q9 $\boldsymbol{\frac{x^3}{a^3}+\frac{3x}{a}+\frac{3a}{x}+\frac{a^3}{x^3}}$

Solution

$$\frac{x^3}{a^3}+\frac{3x}{a}+\frac{3a}{x}+\frac{a^3}{x^3}$$
$$= \left(\frac{x}{a}\right)^3+3\left(\frac{x}{a}\right)^2\left(\frac{a}{x}\right)+3\left(\frac{x}{a}\right)\left(\frac{a}{x}\right)^2+\left(\frac{a}{x}\right)^3$$
As $a^3+3a^2b+3ab^2+b^3=(a+b)^3$
$$= \left(\frac{x}{a}+\frac{a}{x}\right)^3$$

Q10 $\boldsymbol{27a^3+189a^2b+441ab^2+343b^3}$

Solution

$27a^3+189a^2b+441ab^2+343b^3$
$= (3a)^3+3(3a)^2(7b)+3(3a)(7b)^2+(7b)^3$
As $a^3+3a^2b+3ab^2+b^3=(a+b)^3$
$= (3a+7b)^3$

Q11 $8x^3-4x+\frac{2}{3x}-\frac{1}{27x^3}$

Solution

$$8x^3-4x+\frac{2}{3x}-\frac{1}{27x^3}$$
$$= (2x)^3-3(2x)^2\left(\frac{1}{3x}\right)+3(2x)\left(\frac{1}{3x}\right)^2-\left(\frac{1}{3x}\right)^3$$
As $a^3-3a^2b+3ab^2-b^3=(a-b)^3$
$$= \left(2x-\frac{1}{3x}\right)^3$$

Chapter # 5

Exercise# 5.5

Q1 $\boldsymbol{a^3 - 27}$

Solution

$a^3 - 27$

$= (a)^3 - (3)^3$

Using Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (a-3)[(a)^2 + (a)(3) + (3)^2]$

$= (a-3)(a^2 + 3a + 9)$

Q2 $\boldsymbol{a^6 + b^6}$

Solution

$a^6 + b^6$

$= (a^2)^3 + (b^2)^3$

Using Formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (a^2 + b^2)[(a^2)^2 - (a^2)(b^2) + (b^2)^2]$

$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$

Q3 $\boldsymbol{24x^3 + 3}$

Solution

$24x^3 + 3$

$= 3(8x^3 + 1)$

$= 3[(2x)^3 + (1)^3]$

Using Formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= 3\{(2x+1)[(2x)^2 - (2x)(1) + (1)^2]\}$

$= 3(2x+1)(4x^2 + 2x + 1)$

Q4 $\boldsymbol{1 - 27r^3}$

Solution

$1 - 27r^3$

$= (1)^3 - (3r)^3$

Using Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (1-3r)[(1)^2 + (1)(3r) + (3r)^2]$

$= (1-3r)(1 + 3r + 9r^2)$

Q5 $\boldsymbol{2x^3 - 128}$

Solution

$2x^3 - 128$

$2(x^3 - 64)$

$2[(x)^3 - (4)^3]$

Using Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$2\{(x-4)[(x)^2 + (x)(4) + (4)^2]\}$

$2(x-4)(x^2 + 4x + 16)$

Q6 $\boldsymbol{4x^5 - 256x^2}$

Solution

$4x^5 - 256x^2$

$= 4x^2(x^3 - 64)$

$= 4x^2[(x)^3 - (4)^3]$

Using Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= 4x^2\{(x-4)[(x)^2 + (x)(4) + (4)^2]\}$

$= 4x^2(x-4)(x^2 + 4x + 16)$

Q7 $\boldsymbol{18(x-y)^3 - 144(a-b)^3}$

Solution

$18(x-y)^3 - 144(a-b)^3$

$= 18[(x-y)^3 - 8(a-b)^3]$

$= 18[(x-y)^3 - (2(a-b))^3]$

Using Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= \boldsymbol{18[(x-y) - 2(a-b)]\left[(x-y)^2 + (x-y)\big(2(a-b)\big) + \big(2(a-b)\big)^2\right]}$

$= \boldsymbol{18(x - y - 2a + 2b)[(x-y)^2 + 2(x-y)(a-b) + 4(a-b)^2]}$

Q8 $\boldsymbol{x^9 + 1}$

Solution

$x^9 + 1$

$= (x^3)^3 + (1)^3$

Using Formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (x^3 + 1)[(x^3)^2 - (x^3)(1) + (1)^2]$

$= (x+1)[(x)^2 - (x)(1) + (1)^2](x^6 - x^3 + 1)$

Using Formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (2x+1)(x^2 - x + 1)(x^6 - x^3 + 1)$

Q9 $\boldsymbol{a^3 + (c+d)^3}$

Solution

$a^3 + (c+d)^3$

Using Formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= [a + (c+d)][(a)^2 - (a)(c+d) + (c+d)^2]$

$= (a + c + d)[a^2 - a(c+d) + (c+d)^2]$

Q10 $\boldsymbol{27x^3 - y^3}$

Solution

$27x^3 - y^3$

$= (3x)^3 - (y)^3$

Using Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (3x - y)[(3x)^2 + (3x)(y) + (y)^2]$

$= (3x - y)(9x^2 + 3xy + y^2)$

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
\hline
x - 1 & x^3 - 2x^2 - 5x + 6 \\
 & \pm x^3 \mp x^2 \\
\hline
 & -x^2 - 5x + 6 \\
 & \mp x^2 \pm x \\
\hline
 & -6x + 6 \\
 & \mp 6x \pm 6 \\
\hline
 & \text{x}
\end{array}
$$

Here $Q(x) = (x^2 - x - 6)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 2x^2 - 5x + 6 = (x - 1)(x^2 - x - 6)$

$= (x - 1)(x^2 + 2x - 3x - 6)$

$= (x - 1)[x(x + 2) - 3(x + 2)]$

$= (x - 1)(x + 2)(x - 3)$

(ii) $\boldsymbol{x^3 + x^2 - 4x - 4}$

Solution

$P(x) = x^3 + x^2 - 4x - 4$

$Let\ x = -1$

$So\ P(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$

$= -1 + 1 + 4 - 4$

$= 0$

Since $P(x) = 0$, So $x + 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 1$

$$
\begin{array}{r|l}
 & x^2 - 4 \\
\hline
x + 1 & x^3 + x^2 - 4x - 4 \\
 & \pm x^3 \pm x^2 \\
\hline
 & -4x - 4 \\
 & \mp 4x \mp 4 \\
\hline
 & \text{X}
\end{array}
$$

Here $Q(x) = (x^2 - 4)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$

$= (x + 1)[(x)^2 - (2)^2]$

$= (x + 1)(x + 2)(x - 2)$

$\boldsymbol{x^3 - 7x + 6}$

Solution

$P(x) = x^3 - 7x + 6$

$Let\ x = 1$

$So\ P(1) = (1)^3 - 7(1) + 6$

$= 1 - 7 + 6$

$= -6 + 6$

$= 0$

Since $P(x) = 0$, So $x - 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 1$

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
\hline
x - 1 & x^3 - 7x + 6 \\
 & \pm x^3 \qquad \mp x^2 \\
\hline
 & x^2 - 7x + 6 \\
 & \pm x^2 \mp x \\
\hline
 & -6x + 6 \\
 & \mp 6x \pm 6 \\
\hline
 & \text{x}
\end{array}
$$

Here $Q(x) = (x^2 + x - 6)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 7x + 6 = (x - 1)(x^2 + x - 6)$

$= (x - 1)(x^2 - 2x + 3x - 6)$

$= (x - 1)[x(x - 2) + 3(x - 2)]$

$= (x - 1)(x - 2)(x + 3)$

(iv) $\boldsymbol{x^3 - 9x^2 + 23x - 15}$

Solution

$P(x) = x^3 - 9x^2 + 23x - 15$

$Let\ x = 1$

$So\ P(1) = (1)^3 - 9(1)^2 + 23(1) - 15$

$= 1 - 9(1) + 23 - 15$

$= 1 - 9 + 8$

$= -8 + 8$

$= 0$

Since $P(x) = 0$, So $x - 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 1$

Chapter # 5

$$
\begin{array}{r|l}
 & x^2 - 8x + 15 \\ \hline
x - 1 & x^3 - 9x^2 + 23x - 15 \\
 & \pm x^3 \mp x^2 \\ \hline
 & -8x^2 + 23x \\
 & \mp 8x^2 \pm 8x \\ \hline
 & 15x - 15 \\
 & \pm 15x \mp 15 \\ \hline
 & \times
\end{array}
$$

Here $Q(x) = (x^2 - 8x + 15)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 7x + 6 = (x - 1)(x^2 - 8x + 15)$

$= (x - 1)(x^2 - 3x - 5x + 15)$

$= (x - 1)[x(x - 3) - 5(x - 3)]$

$= (x - 1)(x - 3)(x - 5)$

(v) $\boldsymbol{x^3 - 4x^2 - 3x + 18}$

Solution

$P(x) = x^3 - 4x^2 - 3x + 18$

$Let\ x = -2$

$So\ P(-2) = (-2)^3 - 4(-2)^2 - 3(-2) + 18$

$= -8 - 4(4) + 6 + 18$

$= -8 - 16 + 24$

$= -24 + 24$

$= 0$

Since $P(x) = 0$, So $x + 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 2$

$$
\begin{array}{r|l}
 & x^2 - 6x + 9 \\ \hline
x + 2 & x^3 - 4x^2 - 3x + 18 \\
 & \pm x^3 \pm 2x^2 \\ \hline
 & -6x^2 - 3x \\
 & \mp 6x^2 \mp 12x \\ \hline
 & 9x + 18 \\
 & \pm 9x \pm 18 \\ \hline
 & \times
\end{array}
$$

Here $Q(x) = (x^2 - 6x + 9)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 4x^2 - 3x + 18 = (x + 2)(x^2 - 6x + 9)$

$= (x + 2)[(x)^2 - 2(x)(3) + (3)^2]$

$= (x + 2)(x - 3)^2$

(vi) $\boldsymbol{x^3 + 2x^2 - 19x - 20}$

Solution

$P(x) = x^3 + 2x^2 - 19x - 20$

$Let\ x = -1$

$So\ P(-1) = (-1)^3 + 2(-1)^2 - 19(-1) - 20$

$= -1 + 2(1) + 19 - 20$

$= -1 + 2 - 1$

$= 1 - 1$

$= 0$

Since $P(x) = 0$, So $x + 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 1$

$$
\begin{array}{r|l}
 & x^2 + x - 20 \\ \hline
x + 1 & x^3 + 2x^2 - 19x - 20 \\
 & \pm x^3 \pm x^2 \\ \hline
 & x^2 - 19x \\
 & \pm x^2 \pm x \\ \hline
 & -20x - 20 \\
 & \mp 20x \mp 20 \\ \hline
 & \times
\end{array}
$$

Here $Q(x) = (x^2 + x - 20)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 + 2x^2 - 19x - 20 = (x + 1)(x^2 + x - 20)$

$= (x + 1)(x^2 - 4x + 5x - 20)$

$= (x + 1)[x(x - 4) + 5(x - 4)]$

$= (x + 1)(x - 4)(x + 5)$

(vii) $\boldsymbol{x^3 - x^2 - 14x + 24}$

Solution

$P(x) = x^3 - x^2 - 14x + 24$

$Let\ x = 2$

$So\ P(-2) = (2)^3 - (2)^2 - 14(2) + 24$

$= 8 - 4 - 28 + 24$

$= 4 - 4$

$= 0$

Since $P(x) = 0$, So $x - 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 2$

$$
\begin{array}{r|l}
 & x^2 + x - 12 \\
\hline
x - 2 & x^3 - x^2 - 14x + 24 \\
 & \pm x^3 \mp 2x^2 \\
\hline
 & x^2 - 14x \\
 & \pm x^2 \mp 2x \\
\hline
 & -12x + 24 \\
 & \mp 12x \pm 24 \\
\hline
 & \text{x}
\end{array}
$$

Here $Q(x) = (x^2 + x - 12)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - x^2 - 14x + 24 = (x - 2)(x^2 + x - 12)$

$= (x - 2)(x^2 + 4x - 3x - 12)$

$= (x - 2)[x(x + 4) - 3(x + 4)]$

$= (x - 2)(x + 4)(x - 3)$

(viii) $\boldsymbol{x^3 - 6x^2 + 32}$

Solution

$P(x) = x^3 - 6x^2 + 32$

$Let\ x = -2$

$So\ P(-2) = (-2)^3 - 6(-2)^2 + 32$

$= -8 - 6(4) + 32$

$= -8 - 24 + 32$

$= -32 + 32$

$= 0$

Since $P(x) = 0$, So $x + 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 2$

$$
\begin{array}{r|l}
 & x^2 - 8x + 16 \\
\hline
x + 2 & x^3 - 6x^2 + 32 \\
 & \pm x^3 \pm 2x^2 \\
\hline
 & -8x^2 + 32 \\
 & \mp 8x^2 \quad \mp 16x \\
\hline
 & 16x + 32 \\
 & \pm 16x \pm 32 \\
\hline
 & \text{x}
\end{array}
$$

Here $Q(x) = (x^2 - 8x + 16)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$x^3 - 6x^2 + 32 = (x + 2)(x^2 - 8x + 16)$

$= (x + 2)[(x)^2 - 2(x)(4) + (4)^2]$

$= (x + 2)(x - 4)^2$

	Example # 7, 8, 9 Page # 130, 131
	Example # 12 Page + 133
	Example # 17 Page # 136
	Example # 22, 23, 24, 25 Page # 140, 141

Chapter # 6

UNIT # 6

ALGEBRAIC MANIPULATIONS

Ex # 6.1

Highest Common Factor (H.C.F)

The highest number of factors common to all given expressions or polynomials is called Highest Common Factor (H.C.F)

In other words, H.C.F of two or more polynomials is a polynomial of the highest degree, which divides exactly the given polynomials.

There are two methods for finding H.C.F.

(i) H.C.F by Factorization

(ii) H.C.F by Division

H.C.F by Factorization

In this method, first factorize all the given expressions

Then we take all possible common factors which is the H.C.F of the given expression.

Example # 1

Find H.C.F of $x^2 - y^2,\ x^2 - xy$

Solution:

$x^2 - y^2,\ x^2 - xy$

$x^2 - y^2 = (x + y)(x - y)$

And

$x^2 - xy = x(x - y)$

Here $x - y$ is a common factor. Thus

$\mathrm{H.C.F} = \mathrm{x - y}$

Example # 2

Find H.C.F of $ax^2 + 5ax + 6a,$
$ax^3 + 9ax^2 + 14ax \ \ and \ \ 15a(x^2 - 4)$

Solution:

$ax^2 + 5ax + 6a,\ \ ax^3 + 9ax^2 + 14ax\ and$
$15a(x^2 - 4)$

$ax^2 + 5ax + 6a = a(x^2 + 5x + 6)$

$ax^2 + 5ax + 6a = a(x^2 + 2x + 3x + 6)$

$ax^2 + 5ax + 6a = a[x(x + 2) + 3(x + 2)]$

$ax^2 + 5ax + 6a = a(x + 2)(x + 3)$

And

$ax^3 + 9ax^2 + 14ax = ax(x^2 + 9x + 14)$

$ax^3 + 9ax^2 + 14ax = ax(x^2 + 2x + 7x + 14)$

$ax^3 + 9ax^2 + 14ax = ax[x(x + 2) + 7(x + 2)]$

$ax^3 + 9ax^2 + 14ax = ax(x + 2)(x + 7)$

Ex # 6.1

Now also

$15a(x^2 - 4) = 3 \times 5.a[(x)^2 - (2)^2]$

$15a(x^2 - 4) = 3 \times 5.a(x + 2)(x - 2)$

Here $a(x + 2)$ is common in given three expressions.

$\mathrm{H.C.F} = \mathrm{a}(x + 2)$

Note:

The H.C.F $\mathrm{a}(x + 2)$ exactly divides all the given three expression

H.C.F by Division Method

Dividend

$x^2 - x - 6 \,\big)\, x^2 - 2x - 3 \,\big(\, 1$

$\pm x^2 \mp x \mp 6$

$-x + 3$

Remainder

Divisor **Quotient**

Steps

1 Write the expressions in descending order

2 Take the common from the expressions if any.

3 Divide higher degree polynomial by the polynomial of lower degree

4 Divide to that time till the degree of remainder is less than the degree of divisor.

5 Now bring down the divisor and divide by remainder BUT before this take the common from the remainder if any.

6 Repeat the above steps till the remainder is zero.

7 Last divisor is the H.C.F of the given polynomials.

Note:

1 In H.C.F by division, if required, multiply the expression by a suitable integer to avoid fraction.

2 To find the H.C.F of three polynomials, first find H.C.F of any two of them, then find H.C.F of this H.C.F and the third polynomial.

$(x^2)(6) = 6x^2$

Add	Multiply
$+2x$	$+2x$
$+3x$	$+3x$
$\boldsymbol{+5x}$	$\boldsymbol{6x^2}$

$(x^2)(14) = 14x^2$

Add	Multiply
$+2x$	$+2x$
$+7x$	$+7x$
$\boldsymbol{+9x}$	$\boldsymbol{14x^2}$

Chapter # 6

Ex # 6.1

H.C.F by Division method in Urdu

1. تمام variables کو descending order میں لکھیں گے۔
2. اگر کوئی common ہو تو پہلے common لینگے۔
3. بڑے expression کو چھوٹے expression پر divide کریں گے۔
4. اس کو اس وقت تک divide کرتے رہیں گے جب تک remainder میں power ہمارے ساتھ divisor کے power سے کم نہ آئے
5. پھر divisor کو نیچے لائیں گے اور remainder پر divide کریں گے لیکن اس سے پہلے remainder میں common لیں گے اگر ہو۔
6. ان steps کو اس وقت تک کرو گے جب تک remainder میں zero نہ آئے۔
6. آخری divisor ہمارے ساتھ H.C.F ہوگا۔

Example # 3

Find H.C.F of $2x^3 + 7x^2 + 4x - 4$ and $2x^3 + 9x^2 + 11x + 2$

Solution:

$2x^3 + 7x^2 + 4x - 4$ and $2x^3 + 9x^2 + 11x + 2$

$$2x^3 + 7x^2 + 4x - 4 \;\big)\; 2x^3 + 9x^2 + 11x + 2 \;\big(\; 1$$
$$\pm 2x^3 \pm 7x^2 \pm 4x \mp 4$$
$$2x^2 + 7x + 6 \;\big)\; 2x^3 + 7x^2 + 4x - 4 \;\big(\; x$$
$$\pm 2x^3 \pm 7x^2 \pm 6x$$
$$-2 \;\big)\; -2x - 4 \qquad \textbf{Dividing by } -2$$
$$x + 2 \;\big)\; 2x^2 + 7x + 6 \;\big(\; 2x + 3$$
$$\pm 2x^2 \pm 4x$$
$$3x + 6$$
$$\pm 3x \pm 6$$
$$\times$$

Hence H.C.F= $x + 2$

Note:

H.C.F by Factorization

H.C.F of 24 and 32

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 24 = 1, 2, 4, 8, 16, 32

Common factors $= 1, 2, 4, 8$

$\mathrm{H.C.F} = 8$

Chapter # 6

Ex # 6.1

Example # 4

Find H.C.F of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

Solution:

$x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

$$3x^3-5x^2+6x-4 \;\big)\; 3x^3+5x^2-6x-2 \;\big(\; 1$$
$$\pm 3x^3 \mp 5x^2 \pm 6x \mp 4$$
$$2 \;\big|\; 10x^2-12x+2 \qquad \textbf{Dividing by 2}$$
$$5x^2-6x+1 \;\big)\; 3x^3-5x^2+6x-4 \;\big(\; 3x-7$$
$$\times 5 \qquad \textbf{Multiplying by 5}$$
$$15x^3-25x^2+30x-20$$
$$\pm 15x^3 \mp 18x^2 \pm 3x$$
$$-7x^2+27x-20$$
$$\times 5 \qquad \textbf{Multiplying by 5}$$
$$-35x^2+135x-100$$
$$\mp 35x^2 \pm 42x \mp 7$$
$$93 \;\big|\; 93x-93 \qquad \textbf{Dividing by 93}$$
$$x-1 \;\big)\; 5x^2-6x+1 \;\big(\; 5x-1$$
$$\pm 5x^2 \mp 5x$$
$$-x+1$$
$$\mp x \pm 1$$
$$\times$$

Hence H.C.F= $x-1$

Now find the H.C.F of $x-1$ and $x^3-6x^2+11x-6$

$$x-1 \;\big)\; x^3-6x^2+11x-6 \;\big(\; x^2-5x+6$$
$$\pm x^3 \mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

Hence the required H.C.F of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$ is $\boldsymbol{x-1}$

Least Common Multiple (L.C.M)

The polynomial of least degree which is divisible by the given polynomials.

There are two methods of finding L.C.M

(a) L.C.M by factorization

(b) L.C.M by formula

Chapter # 6

Ex # 6.1

(a) **L.C.M by factorization**

In this method, first factorize all the given expressions

Then find the L.C.M by given formula.

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

Example # 5

Find L.C.M of $x^2 + 4x + 4$ and $x^2 + 5x + 6$

Solution:

$x^2 + 4x + 4$ and $x^2 + 5x + 6$

$x^2 + 4x + 4 = (x)^2 + 2(x)(2) + (2)^2$

$x^2 + 4x + 4 = (x + 2)^2$

$x^2 + 4x + 4 = (x + 2)(x + 2)$

Now

$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

$x^2 + 5x + 6 = x(x + 2) + 3(x + 2)$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$Common\ Factor = x + 2$

$Non - common\ factor = (x + 2)(x + 3)$

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x + 2)(x + 2)(x + 3)$

$L.C.M = (x + 2)^2(x + 3)$

Example # 6

Find L.C.M of $x^2 - 4x + 3$, $x^2 - 3x + 2$ and $x^2 - 5x + 6$

Solution:

$x^2 - 4x + 3$, $x^2 - 3x + 2$ and $x^2 - 5x + 6$

$x^2 - 4x + 3 = x^2 - x - 3x + 3$

$x^2 - 4x + 3 = x(x - 1) - 3(x - 1)$

$x^2 - 4x + 3 = (x - 1)(x - 3)$….. (i)

Now

$x^2 - 3x + 2 = x^2 - x - 2x + 3$

$x^2 - 3x + 2 = x(x - 1) - 2(x - 1)$

$x^2 - 3x + 2 = (x - 1)(x - 2)$….. (ii)

Now

$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$x^2 - 5x + 6 = x(x - 2) - 3(x + 2)$

$x^2 - 5x + 6 = (x - 2)(x - 3)$….. (iii)

$x - 1$ in expression (i)& (ii)

$x - 2$ in expression (ii)& (iii)

$x - 3$ in expression (i)& (iii)

Therefore:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x - 1)(x - 2)(x - 3) \times 1$

$L.C.M = (x - 1)(x - 2)(x - 3)$

Ex # 6.1

L.C.M Theorem:

If A and B are given polynomials and their H.C.F and L.C.M are represented by H and L respectively, then

$$A \times B = H \times L$$

Proof:

Since H is common factor of polynomial of A and B, then it divides exactly A and B. So

$\frac{A}{H} = a$

$A = Ha$… equ(i)

and

$\frac{B}{H} = b$

$B = Hb$… equ(ii)

As a and b have no common factor.

As we know that:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L = H \times a \times b$

Multiply B.S by H

$L \times H = H \times a \times b \times H$

$L \times H = (Ha) \times (Hb)$

Put equ(i) and equ(ii), we get

$L \times H = A \times B$

Or

$H \times L = Product\ of\ two\ polynomials$

Formula for L.C.M

As $L \times H = A \times B$

$L = \frac{A \times B}{H}$

$L.C.M = \frac{Product\ of\ two\ polynomials}{H.C.F}$

Chapter # 6

Ex # 6.1

Example # 7

Find L.C.M of $x^3 - 6x^2 + 11x - 6$ and $x^3 - 4x + 3$

$x^3 - 6x^2 + 11x - 6$ and $x^3 - 4x + 3$

Solution:

$Let\ A = x^3 - 6x^2 + 11x - 6$

$and\ B = x^3 - 4x + 3$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} -- equ\ (i)$$

First we find H.C.F

$$x^3 - 4x + 3 \overline{)\, x^3 - 6x^2 + 11x - 6 \,}\ 1$$

$\pm x^3 \mp 4x \pm 3$

$-3\ |\ -6x^2 + 15x - 9$

$2x^2 - 5x + 3\ \overline{)\, x^3 - 4x + 3 \,}\ x + 5$

$\times 2$

$2x^3 - 8x + 6$

$\pm 2x^3 \pm 3x \mp 5x^2$

$5x^2 - 11x + 6$

$\times 2$

$10x^2 - 22x + 12$

$\pm 10x^2 \mp 25x \pm 15$

$3\ |\ 3x - 3$

$x - 1\ \overline{)\, 2x^2 - 5x + 3 \,}\ 2x - 3$

$\pm 2x^2 \mp 2x$

$-3x + 3$

$\mp 3x \pm 3$

$\times$

$H.C.F = x - 1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3 - 6x^2 + 11x - 6)(x^3 - 4x + 3)}{x - 1}$$

Now by Simple Division

$$x - 1 \overline{)\, x^3 - 6x^2 + 11x - 6 \,} \quad x^2 - 5x + 6$$

$\pm x^3 \mp x^2$

$-5x^2 + 11x - 6$

$\mp 5x^2 \pm 5x$

$6x - 6$

$\pm 6x \mp 6$

$\times$

$So\ L.C.M = (x^2 - 5x + 6)(x^3 - 4x + 3)$

Chapter # 6

Ex # 6.1

Example # 8

Find H.C.F and L.C.M of $3x^3 - 2x^2 - 3x + 2$ and $6x^3 - 7x^2 - x + 2$

$3x^3 - 2x^2 - 3x + 2$ and $6x^3 - 7x^2 - x + 2$

Solution:

$Let\ A = 3x^3 - 2x^2 - 3x + 2$

$and\ B = 6x^3 - 7x^2 - x + 2$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$3x^3 - 2x^2 - 3x + 2 \overline{\big) \; 6x^3 - 7x^2 - x + 2 \;} \; 2$$

$$\underline{\pm 6x^3 \mp 4x^2 \mp 6x \pm 4}$$

$$-1 \overline{\big) \; -3x^2 + 5x - 2}$$

$$3x^2 - 5x + 2 \overline{\big) \; 3x^3 - 2x^2 - 3x + 2 \;} \; x + 1$$

$$\underline{\pm 3x^3 \mp 5x^2 \pm 2x}$$

$$3x^2 - 5x + 2$$

$$\underline{\pm 3x^2 \mp 5x \pm 2}$$

$$\times$$

$$H.C.F = 3x^2 - 5x + 2$$

Now put the values in equ (i)

$$L.C.M = \frac{(3x^3 - 2x^2 - 3x + 2)(6x^3 - 7x^2 - x + 2)}{3x^2 - 5x + 2}$$

Now by Simple Division

$$\begin{array}{r|l} & x + 1 \\ 3x^2 - 5x + 2 & 3x^3 - 2x^2 - 3x + 2 \\ & \underline{\pm 3x^3 \mp 5x^2 \pm 2x} \\ & \quad 3x^2 - 5x + 2 \\ & \underline{\pm 3x^2 \mp 5x \pm 2} \\ & \quad \times \end{array}$$

$So\ L.C.M = (x + 1)(6x^3 - 7x^2 - x + 2)$

Example # 9

If H.C.F and L.C.M of two polynomials are $x - 3$ and $x^3 - 9x^2 + 26x - 24$ respectively. Find the second polynomial when one polynomial is $x^2 - 5x + 6$.

Solution:

$H.C.F = x - 3$

$L.C.M = x^3 - 9x^2\_26x - 24$

$Let\ First\ polynomial = A = x^2 - 5x + 6$

$Second\ polynomial = B = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

$A \times B = L.C.M \times H.C.F$

Chapter # 6

Ex # 6.1

$$B = \frac{L.C.M \times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3 - 9x^2\_26x - 24)(x-3)}{x^2 - 5x + 6}$$

Now by simple Division

$$\begin{array}{r|l} & \quad x - 4 \\ x^2 - 5x + 6 & x^3 - 9x^2 + 26\,x - 24 \\ & \pm x^3 \mp 5x^2 \pm 6x \\ \hline & \quad -4x^2 + 20x - 24 \\ & \quad \mp 4x^2 \pm 20x \mp 24 \\ \hline & \qquad \times \end{array}$$

$So\ B = (x-4)(x-3)$
$B = x^2 - 3x - 4x + 12$
$B = x^2 - 7x + 12$
Hence the second polynomial is $x^2 - 7x + 12$

Example # 10

If H.C.F and L.C.M of two polynomials are $x - 1$ and $x^3 + 4x^2 + x - 6$ respectively. Find the polynomials of degree 2.

Solution:

$H.C.F = x - 1$
$L.C.M = x^3 + 4x^2 + x - 6$
$First\ polynomial = A = ?$
$Second\ polynomial = B = ?$
$As\ H.C.F = x - 1$
$then\ it\ is\ also\ the\ factor\ of\ L.C.M$
Now

$$\begin{array}{r|l} & \quad x^2 + 5x + 6 \\ x - 1 & x^3 + 4x^2 + x - 6 \\ & \pm x^3 \mp x^2 \\ \hline & \quad 5x^2 + x - 6 \\ & \pm 5x^2 \mp 5x \\ \hline & \quad 6x - 6 \\ & \quad \pm 6x \mp 6 \\ \hline & \qquad \times \end{array}$$

$L.C.M = x^3 + 4x^2 + x - 6$
$L.C.M = (x-1)(x^2 + 5x + 6)$
$L.C.M = (x-1)(x^2 + 3x + 2x + 6)$
$L.C.M = (x-1)[x(x+3) + 2(x+3)]$
$L.C.M = (x-1)(x+3)(x+2)$
$As\ x - 1\ is\ common\ factor. So$
$A = (x-1)(x+3)$

Ex # 6.1

$A = x^2 + 2x - 3$
And
$B = (x-1)(x+2)$
$B = x^2 + 2x - 1x - 2$
$B = x^2 + x - 2$

Example # 11

The sum of two numbers is 120 and their H.C.F is 12. Find the numbers.

Solution:

Let x and y be the two numbers.
As H.C.F is 12, means 12 is common factor.
So, it becomes
$12x + 12y = 120$
$12(x + y) = 120$
Divide B.S by 12, we get
$x + y = 12$
As the sum of two numbers is 10, so the possible pairs of numbers are (1,9), (2,8), (3,7), (4,6), (5,5)
As (1,9), (3,7) are non commo factors
Then the required numbers are:
$1 \times 12 = 12$ and $9 \times 12 = 108$
OR
$3 \times 12 = 36$ and $7 \times 12 = 84$

Chapter # 6

Exercise# 6.1

Page # 159-160

Q1: 159 **Find H.C.F of the following expression by factorization method.**

(i) $(x+y)^2$ **and** x^2-36

Solution:

$(x+y)^2$ and x^2-36

$(x+y)^2=(x+y)(x+y)$

And

$x^2-36=(x)^2-(6)^2$

$=(x+6)(x-6)$

$H.C.F=x-6$

(iii) $x-3, x^2-9, (x-3)^2$

Solution:

$x-3, x^2-9, (x-3)^2$

$x-3=x-3$

And

$x^2-9=(x)^2-(3)^2$

$=(x+3)(x-3)$

And

$(x-3)^2=(x-3)(x-3)$

$H.C.F= x-3$

(ii) x^4-y^4 **and** $x^4+2x^2y^2+y^4$

Solution:

x^4-y^4 and $x^4+2x^2y^2+y^4$

$x^4-y^4=(x^2)^2-(y^2)^2$

$=(x^2+y^2)(x^2-y^2)$

$=(x^2+y^2)(x+y)(x-y)$

And

$x^4+2x^2y^2+y^4=(x^2)^2+2(x^2)(y^2)+(y^2)^2$

$=(x^2+y^2)^2$

$=(x^2+y^2)(x^2+y^2)$

$H.C.F=x^2+y^2$

(v) $2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

Solution:

$2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

$2x^4-2y^4=2[(x^2)^2-(y^2)^2]$

$=2(x^2+y^2)(x^2-y^2)$

$=2(x^2+y^2)(x+y)(x-y)$

And

$6x^2+12xy+6y^2=6(x^2+2xy+y^2)$

$=2\times 3(x+y)^2$

$=2\times 3(x+y)(x+y)$

And

$9x^3+9y^3=9(x^3+y^3)$

$=9(x+y)(x^2-xy+y^2)$

$H.C.F=x+y$

(iv) $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

Solution:

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2=2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3=2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y)=3.3(x-y)(x-y)(x+2y)$

$H.C.F=3.3(x-y)(x-y)(x+2y)$

$H.C.F=3^2(x-y)^2(x+2y)$

Chapter # 6

Ex # 6.1

Q2: Find H.C.F by division method.

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(i) $\boldsymbol{x^2 - x - 6}$ **and** $\boldsymbol{x^2 - 2x - 3}$

Solution:

$x^2 - x - 6$ and $x^2 - 2x - 3$

$$
\begin{array}{r|l|l}
x^2 - x - 6 & x^2 - 2x - 3 & 1 \\
 & \pm x^2 \mp x \mp 6 & \\
\hline
-1 & -x + 3 & \\
\end{array}
$$

$$
\begin{array}{r|l|l}
x - 3 & x^2 - x - 6 & x + 2 \\
 & \pm x^2 \mp 3x & \\
\hline
 & 2x - 6 & \\
 & \pm 2x \mp 6 & \\
\hline
 & \times & \\
\end{array}
$$

$H.C.F = x - 3$

(ii) $\boldsymbol{y^3 - 3y + 2}$ **and** $\boldsymbol{y^3 - 5y^2 + 7y - 3}$

Solution:

$y^3 - 3y + 2$ and $y^3 - 5y^2 + 7y - 3$

$$
\begin{array}{r|l|l}
y^3 - 3y + 2 & y^3 - 5y^2 + 7y - 3 & 1 \\
 & \pm y^3 \qquad \mp 3y \pm 2 & \\
\hline
-5 & -5y^2 + 10y - 5 & \\
\end{array}
$$

$$
\begin{array}{r|l|l}
y^2 - 2y + 1 & y^3 - 3y + 2 & y + 2 \\
 & \pm y^3 \pm 1y \quad \mp 2y^2 & \\
\hline
 & 2y^2 - 4y + 2 & \\
 & \pm 2y^2 \mp 4y \pm 2 & \\
\hline
 & \times & \\
\end{array}
$$

$H.C.F = y^2 - 2y + 1$

Chapter # 6

Ex # 6.1

(iii) $\mathbf{2x^5 - 4x^4 - 6x}$ **and** $\mathbf{x^5 + x^4 - 3x^3 - 3x^2}$

Solution:

$2x^5 - 4x^4 - 6x$ and $x^5 + x^4 - 3x^3 - 3x^2$

$2x^5 - 4x^4 - 6x = 2x(x^4 - 2x^3 - 3)$

$x^5 + x^4 - 3x^3 - 3x^2 = x^2(x^3 + x^2 - 3x - 3)$

$= x.x(x^3 + x^2 - 3x - 3)$

$x^3 + x^2 - 3x - 3 \;\big)\; x^4 - 2x^3 - 3 \;\big(\; x$

$\pm x^4 \pm x^3 \quad \mp 3x^2 \mp 3x$

$-3 \;\big|\; -3x^3 + 3x^2 + 3x - 3$

$x^3 - x^2 - x + 1 \;\big)\; x^3 + x^2 - 3x - 3 \;\big(\; 1$

$\pm x^3 \mp x^2 \mp x \pm 1$

$2 \;\big|\; 2x^2 - 2x - 4$

$x^2 - x - 2 \;\big)\; x^3 - x^2 - x + 1 \;\big(\; x$

$\pm x^3 \mp x^2 \mp 2x$

$x + 1 \;\big)\; x^2 - x - 2 \;\big(\; x - 2$

$\pm x^2 \pm x$

$-2x - 2$

$\mp 2x \mp 2$

$\times$

$H.C.F = x(x + 1)$

(iv) $\mathbf{2x^3 + 10x^2 + 5x + 25}$ **and** $\mathbf{x^3 + 5x^2 - x - 5}$

Solution:

$2x^3 + 10x^2 + 5x + 25$ and $x^3 + 5x^2 - x - 5$

$x^3 + 5x^2 - x - 5 \;\big)\; 2x^3 + 10x^2 + 5x + 25 \;\big(\; 2$

$\pm 2x^3 \pm 10x^2 \mp 2x \mp 10$

$7 \;\big|\; 7x + 35$

$x + 5 \;\big)\; x^3 + 5x^2 - x - 5 \;\big(\; x^2 - 1$

$\pm x^3 \mp 5x^2$

$-x - 5$

$\mp x \mp 5$

$\times$

$H.C.F = x + 5$

Chapter # 6

Ex # 6.1

Q3: Find L.C.M by factorization.

(i) $x+y, \quad x^2-y^2$

Solution:

$x+y, \quad x^2-y^2$

$x+y=x+y$

And

$x^2-y^2=(x+y)(x-y)$

$Common\ Factor=x+y$

$Non-common\ factor=x-y$

$L.C.M=common\ factor\times non-common\ factor$

$L.C.M=(x+y)(x-y)$

$L.C.M=x^2-y^2$

(ii) $x^3-y^3, x-y$

Solution:

$x^3-y^3, x-y$

$x^3-y^3=(x-y)(x^2+xy+y^2)$

And

$x-y=x-y$

$Common\ Factor=x-y$

$Non-common\ factor=x^2+xy+y^2$

$L.C.M=common\ factor\times non-common\ factor$

$L.C.M=(x-y)(x^2+xy+y^2)$

$L.C.M=x^3-y^3$

Ex # 6.1

(iii) $x^5-x, x^5-x^2\ and\ x^5-x^3$

Solution:

$x^5-x, x^5-x^2\ and\ x^5-x^3$

$x^5-x=x(x^4-1)$

$=x[(x^2)^2-(1)^1]$

$=x(x^2+1)(x^2-1)$

$=x(x^2+1)(x+1)(x-1)$

And

$x^5-x^2=x^2(x^3-1)$

$=x.x[(x)^3-(1)^3]$

$=x.x(x-1)(x^2+(x)(1)+1^2)$

$=x.x(x-1)(x^2+x+1)$

And

$x^5-x^3=x^3(x^2-1)$

$=x.x.x[(x)^2-(1)^2]$

$=x.x.x(x+1)(x-1)$

$Common\ Factor=x(x-1)$

$Non-common\ factor=\ x.x(x^2+1)(x+1)(x^2+x+1)$

$L.C.M=common\ factor\times non-common\ factor$

$L.C.M=x(x-1)\times x.x(x^2+1)(x+1)(x^2+x+1)$

$L.C.M=x^3(x-1)(x+1)(x^2+1)(x^2+x+1)$

(iv) $2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

Solution:

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2=2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3=2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y)=3.3(x-y)(x-y)(x+2y)$

$Common\ Factor=3.3(x-y)(x-y)(x+2y)$

$Non-common\ factor=2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M=common\ factor\times non-common\ factor$

$L.C.M=3.3(x-y)(x-y)(x+2y)\times 2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M=2^3 3^2(x-y)^3(x+2y)^3$

Chapter # 6

Ex # 6.1

Q4: 160 **Find H.C.F and L.C.M of the following expression.**

(i) $\boldsymbol{x^3 - 2x^2 - 13x - 10}$ **and** $\boldsymbol{x^3 - x^2 - 10x - 8}$

Solution:

$x^3 - 2x^2 - 13x - 10$ and $x^3 - x^2 - 10x - 8$

$Let\ A = x^3 - 2x^2 - 13x - 10$

$and\ B = x^3 - x^2 - 10x - 8$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$x^3 - x^2 - 10x - 8 \,\big)\, x^3 - 2x^2 - 13x - 10 \,\big(\, 1$$
$$\pm x^3 \mp x^2 \mp 10x \mp 8$$
$$-1 \,\big)\, -x^2 - 3x - 2$$
$$x^2 + 3x + 2 \,\big)\, x^3 - x^2 - 10x - 8 \,\big(\, x - 4$$
$$\pm x^3 \pm 3x^2 \pm 2x$$
$$-4x^2 - 12x - 8$$
$$\mp 4x^2 \mp 12x \mp 8$$
$$\times$$

$H.C.F = x^2 + 3x + 2$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3 - 2x^2 - 13x - 10)(x^3 - x^2 - 10x - 8)}{x^2 + 3x + 2}$$

Now by Simple Division

$$x - 5$$
$$x^2 + 3x + 2 \,\big)\, x^3 - 2x^2 - 13x - 10$$
$$\pm x^3 \pm 3x^2 \pm 2x$$
$$-5x^2 - 15x - 10$$
$$\mp 5x^2 \mp 15x \mp 10$$
$$\times$$

$So\ L.C.M = (x - 5)(x^3 - x^2 - 10x - 8)$

Chapter # 6

Ex # 6.1

(ii) $\boldsymbol{2x^4 - 2x^3 + x^2 + 3x - 6}$ **and** $\boldsymbol{4x^4 - 2x^3 + 3x - 9}$

Solution:

$2x^4 - 2x^3 + x^2 + 3x - 6 \; and \; 4x^4 - 2x^3 + 3x - 9$

$Let\ A = 2x^4 - 2x^3 + x^2 + 3x - 6$

$and\ B = 4x^4 - 2x^3 + 3x - 9$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$2x^4 - 2x^3 + x^2 + 3x - 6 \;\overline{)\; 4x^4 - 2x^3 + 3x - 9 \;}\; 2$$
$$\pm 4x^4 \mp 4x^3 \pm 6x \mp 12 \pm 2x^2$$
$$2x^3 - 2x^2 - 3x + 3 \;\overline{)\; 2x^4 - 2x^3 + x^2 + 3x - 6 \;}\; x$$
$$\pm 2x^4 \mp 2x^3 \mp 3x^2 \pm 3x$$
$$2 \;|\; 4x^2 - 6$$
$$2x^2 - 3 \;\overline{)\; 2x^3 - 2x^2 - 3x + 3 \;}\; x - 1$$
$$\pm 2x^3 \qquad \mp 3x$$
$$-2x^2 + 3$$
$$\mp 2x^2 \pm 3$$
$$\times$$

$H.C.F = 2x^2 - 3$

Now put the values in equ (i)

$$L.C.M = \frac{(2x^4 - 2x^3 + x^2 + 3x - 6)(4x^4 - 2x^3 + 3x - 9)}{2x^2 - 3}$$

Now by Simple Division

$$\begin{array}{r}
x^2 - x + 2 \\
2x^2 - 3 \;\overline{)\; 2x^4 - 2x^3 + x^2 + 3x - 6} \\
\pm 2x^4 \qquad \mp 3x^2 \qquad\qquad \\
\hline
-2x^3 + 4x^2 + 3x - 6 \\
\mp 2x^3 \qquad \pm 3x \qquad \\
\hline
4x^2 - 6 \\
\pm 4x^2 \mp 6 \\
\hline
\times
\end{array}$$

$So\ L.C.M = (x^2 - x + 2)(4x^4 - 2x^3 + 3x - 9)$

Chapter # 6

Ex # 6.1

(iii) $\boldsymbol{a^4 - a^3 - a + 1}$ **and** $\boldsymbol{a^4 + a^2 + 1}$

Solution:

$a^4 - a^3 - a + 1$ and $a^4 + a^2 + 1$

$Let\ A = a^4 - a^3 - a + 1$

$and\ B = a^4 + a^2 + 1$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
a^4 + a^2 + 1 & a^4 - a^3 - a + 1 & 1 \\
 & \pm a^4 \qquad \pm 1 \pm a^2 & \\
\hline
-a & -a^3 - a^2 - a & \\
\hline
a^2 + a + 1 & a^4 + a^2 + 1 & a^2 - a + 1 \\
 & \pm a^4 \pm a^2 \qquad \pm a^3 & \\
\hline
 & -a^3 + 1 & \\
 & \mp a^3 \qquad \mp a^2 \mp a & \\
\hline
 & a^2 + a + 1 & \\
 & \pm a^2 \pm a \pm 1 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = a^2 + a + 1$

Now put the values in equ (i)

$$L.C.M = \frac{(a^4 - a^3 - a + 1)(a^4 + a^2 + 1)}{a^2 + a + 1}$$

Now by Simple Division

$$\begin{array}{r|l}
 & a^2 - 2a + 1 \\
\hline
a^2 + a + 1 & a^4 - a^3 - a + 1 \\
 & \pm a^4 \pm a^3 \qquad \pm a^2 \\
\hline
 & -2a^3 - a^2 - a + 1 \\
 & \mp 2a^3 \mp 2a^2 \mp 2a \\
\hline
 & a^2 + a + 1 \\
 & \pm a^2 \pm a \pm 1 \\
\hline
 & \times
\end{array}$$

$So\ L.C.M = (a^2 - 2a + 1)(a^4 + a^2 + 1)$

Chapter # 6

Ex # 6.1

(iv) $\mathbf{1 - x^2 - x^4 + x^5}$ **and** $\mathbf{1 + 2x + x^2 - x^4 - x^5}$

Solution:

$1 - x^2 - x^4 + x^5$ and $1 + 2x + x^2 - x^4 - x^5$

$x^5 - x^4 - x^2 + 1$ and $-x^5 - x^4 + x^2 + 2x + 1$

$Let\ A = x^5 - x^4 - x^2 + 1$

$and\ B = -x^5 - x^4 + x^2 + 2x + 1$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} --equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
x^5 - x^4 - x^2 + 1 & -x^5 - x^4 + x^2 + 2x + 1 & -1 \\
 & \mp x^5 \pm x^4 \pm x^2 \qquad \mp 1 & \\
\hline
-2 & -2x^4 + 2x + 2 & \\
\hline
 & x^4 - x - 1 &
\end{array}$$

$$\begin{array}{r|l|l}
x^4 - x - 1 & x^5 - x^4 - x^2 + 1 & x - 1 \\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x & \\
\hline
 & -x^4 + x + 1 & \\
 & \mp x^4 \pm x \pm 1 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = \quad x^4 - x - 1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^5 - x^4 - x^2 + 1)(-x^5 - x^4 + x^2 + 2x + 1)}{x^4 - x - 1}$$

Now by Simple Division

$$\begin{array}{r|l}
 & x - 1 \\
\hline
x^4 - x - 1 & x^5 - x^4 - x^2 + 1 \\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x \\
\hline
 & -x^4 + x + 1 \\
 & \mp x^4 \pm x \pm 1 \\
\hline
 & \times
\end{array}$$

$So\ L.C.M = (x + 2)(-x^5 - x^4 + x^2 + 2x + 1)$

$So\ L.C.M = (x + 2)(1 + 2x + x^2 - x^4 - x^5)$

Chapter # 6

Q5: 160

H.C.F and L.C.M of two polynomials are $x-2$ and x^3+3x^2-6x-8 respectively. If one polynomial is x^2+2x-8, find the second polynomial.

Solution:

$H.C.F = x-2$

$L.C.M = x^3+3x^2-6x-8$

$First\ polynomial = A = x^2+2x-8$

$Second\ polynomial = B = ?$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F}$$

$$L.C.M\times H.C.F = A\times B$$

$$\frac{L.C.M\times H.C.F}{A} = B$$

$$B = \frac{L.C.M\times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3+3x^2-6x-8)(x-2)}{x^2+2x-8}$$

Now by simple Division

$$\begin{array}{r|l} & x+1 \\ \hline x^2+2x-8, & x^3+3x^2-6x-8 \\ & \pm x^3\pm 2x^2\mp 8x \\ \hline & x^2+2x-8 \\ & \pm x^2\pm 2x\mp 8 \\ \hline & \times \end{array}$$

$So\ B = (x+1)(x-2)$

$B = x^2-2x+1x-2$

$B = x^2-x-2$

Q6: 160

If product of two polynomials is $x^4+5x^3-6x^2-2x-28$ and their H.C.F is $x-2$. Find their L.C.M.

Solution:

Let Product of two polynomials $= A\times B$

$Then\ A\times B = x^4+5x^3-6x^2-2x-28$

$H.C.F = x-2$

$L.C.M = ?$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F}$$

Put the values

$$L.C.M = \frac{x^4+5x^3-6x^2-2x-28}{x-2}$$

$$\begin{array}{r|l} & x^3+7x^2+8x+14 \\ \hline x-2 & x^4+5x^3-6x^2-2x-28 \\ & \pm x^4\mp 2x^3 \\ \hline & 7x^3-6x^2-2x-28 \\ & \pm 7x^3\mp 14x^2 \\ \hline & 8x^2-2x-28 \\ & \pm 8x^2\mp 16x \\ \hline & 14x-28 \\ & \pm 14\mp 28 \\ \hline & \times \end{array}$$

$L.C.M = x^3+7x^2+8x+14$

Q7: 160

H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find the polynomials of degree 2.

Solution:

$H.C.F = x+5$

$L.C.M = 2x^3+11x^2+2x-15$

$First\ polynomial = A = ?$

$Second\ polynomial = B = ?$

$As\ H.C.F = x+5$

$then\ it\ is\ also\ the\ factor\ of\ L.C.M$

Now

$$\begin{array}{r|l} & 2x^2+x-3 \\ \hline x+5 & 2x^3+11x^2+2x-15 \\ & \pm 2x^3\pm 10x^2 \\ \hline & x^2+2x-15 \\ & \pm x^2\pm 5x \\ \hline & -3x-15 \\ & \mp 3x\mp 15 \\ \hline & \times \end{array}$$

$L.C.M = 2x^3+11x^2+2x-15$

$L.C.M = (x+5)(2x^2+x-3)$

$L.C.M = (x+5)(2x^2+3x-2x-3)$

$L.C.M = (x+5)[x(2x+3)-1(2x+3)]$

$L.C.M = (x+5)(2x+3)(x-1)$

$As\ x+5\ is\ common\ factor. So$

$A = (x+5)(2x+3)$

$A = 2x^2+3x+10x+15$

$A = 2x^2+13x+15$

And

$B = (x+5)(x-1)$

$B = x^2-1x+5x-5$

$B = x^2+4x-5$

Chapter # 6

Ex # 6.1

Q8: 160 **If product of two polynomials is $x^4+6x^3-3x^2-56x-48$ and their L.C.M is $x^3+2x^2-11x-12$. Find their H.C.F.**

Solution:

Let Product of two polynomials $= A \times B$

$Then\ A \times B = x^4+6x^3-3x^2-56x-48$

$L.C.M = x^3+2x^2-11x-12$

$H.C.F = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

$$H.C.F = \frac{A \times B}{L.C.M}$$

Put the values

$$H.C.F = \frac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12}$$

Now by Simple Division

$$\begin{array}{r|l} & x+4 \\ x^3+2x^2-11x-12 & x^4+6x^3-3x^2-56x-48 \\ & \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\ \hline & 4x^3+8x^2-44x-48 \\ & \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\ \hline & \times \end{array}$$

$So\ H.C.F = x+4$

Q9: 160 **Waqar wishes to distribute 128 bananas and also 176 apples equally among certain number of children. Find the highest number of children who can get the fruit in this way.**

Solution:

Bananas = 128

Apples = 176

Highest number of children = ?

Now

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	176
2	88
2	44
2	22
11	11
	1

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$176 = 2 \times 2 \times 2 \times 2 \times 11$

$H.C.F = 2 \times 2 \times 2 \times 2$

$= 16$

So highest number of children = 16

Ex # 6.2

Algebraic fractions

An algebraic fraction is the quotient of two algebraic expressions.

Example:

$$\frac{x-y}{y^2-4x^2}$$

Example # 12

Simplify $\dfrac{x+y}{3x+2y}+\dfrac{x-y}{3x+2y}$

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$

$$=\frac{x+y+x-y}{3x+2y}$$

$$=\frac{x+x+y-y}{3x+2y}$$

$$=\frac{2x}{3x+2y}$$

Example # 13

Simplify $\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{x^2-y^2}$

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$

$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$

$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$

$$=\frac{3y^2-2xy}{x^2-y^2}$$

Chapter # 6

Ex # 6.2

Example # 14

Simplify $\dfrac{x^2-xy+y^2}{x^3+y^3}+\dfrac{x^2+xy+y^2}{x^3-y^3}-\dfrac{1}{x^2-y^2}$

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$$

$$=\frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}+\frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$=\frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$=\frac{x+x-y+y-1}{x^2-y^2}$$

$$=\frac{2x-1}{x^2-y^2}$$

Example # 15

Simplify $\dfrac{y}{y^2-y-2}-\dfrac{1}{y^2+5y-14}-\dfrac{2}{y^2+8y+7}$

Solution:

$$\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$$

$$=\frac{y}{y^2-2y+y-2}-\frac{1}{y^2-2y+7y-14}-\frac{2}{y^2+1y+7y+7}$$

$$=\frac{y}{y(y-2)+1(y-2)}-\frac{1}{y(y-2)+7(y-2)}-\frac{2}{y(y+1)+7(y+1)}$$

$$=\frac{y}{(y-2)(y+1)}-\frac{1}{(y-2)(y+7)}-\frac{2}{(y+1)(y+7)}$$

$$=\frac{y(y+7)-1(y+1)-2(y-2)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+7y-1y-1-2y+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+6y-2y-1+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+4y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+1y+3y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y(y+1)+3(y+1)}{(y-2)(y+1)(y+7)}$$

$$=\frac{(y+1)(y+3)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y+3}{(y-2)(y+7)}$$

Ex # 6.2

Example # 16

Simplify $\dfrac{x+4}{x-3}\times\dfrac{x^2-9}{x^2-x-2}$

Solution:

$$\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$$

$$=\frac{x+4}{x-3}\times\frac{x^2-3^2}{x^2-2x+1x-2}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{x(x-2)+1(x-2)}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{(x-2)(x+1)}$$

$$=\frac{x+4}{1}\times\frac{(x+3)}{(x-2)(x+1)}$$

$$=\frac{(x+4)(x+3)}{(x-2)(x+1)}$$

Example # 17

Multiply $\dfrac{x^2-2x}{2x^2+5x+3}$ **by** $\dfrac{2x^2-3x-9}{x^2-9}$

Solution:

$$\frac{x^2-2x}{2x^2+5x+3}\times\frac{2x^2-3x-9}{x^2-9}$$

$$=\frac{x(x-2)}{2x^2+2x+3x+3}\times\frac{2x^2+3x-6x-9}{x^2-9^2}$$

$$=\frac{x(x-2)}{2x(x+1)+3(x+1)}\times\frac{x(2x+3)-3(2x+3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)(2x+3)}\times\frac{(2x+3)(x-3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)}\times\frac{1}{(x+3)}$$

$$=\frac{x(x-2)}{(x+1)(x+3)}$$

Example # 18

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

Solution:

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

$$=\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\times\frac{y^2}{x^2+xy+y^2}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{y.y.y}\times\frac{y}{x-y}\times\frac{y.y}{x^2+xy+y^2}$$

$$=1$$

Chapter # 6

Ex # 6.2

Q1: Simplify:

(*i*) $\dfrac{\boldsymbol{x}}{\boldsymbol{x+y}}+\dfrac{\boldsymbol{2y}}{\boldsymbol{x+y}}$

Solution:

$$\frac{x}{x+y}+\frac{2y}{x+y}$$
$$=\frac{x+2y}{x+y}$$

(*ii*) $\dfrac{\boldsymbol{x+y}}{\boldsymbol{3x+2y}}+\dfrac{\boldsymbol{x-y}}{\boldsymbol{3x+2y}}$

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$
$$=\frac{x+y+x-y}{3x+2y}$$
$$=\frac{x+x+y-y}{3x+2y}$$
$$=\frac{2x}{3x+2y}$$

(*iii*) $\dfrac{\boldsymbol{3}}{\boldsymbol{y-2}}-\dfrac{\boldsymbol{2}}{\boldsymbol{y+2}}-\dfrac{\boldsymbol{y}}{\boldsymbol{y^2-4}}$

Solution:

$$\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-(2)^2}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{(y+2)(y-2)}$$
$$=\frac{3(y+2)-2(y-2)-y}{(y+2)(y-2)}$$
$$=\frac{3y+6-2y+4-y}{(y+2)(y-2)}$$
$$=\frac{3y-2y-y+6+4}{(y+2)(y-2)}$$
$$=\frac{3y-3y+10}{y^2-(2)^2}$$
$$=\frac{10}{y^2-4}$$

(*iv*) $\dfrac{\boldsymbol{x-y}}{\boldsymbol{x+y}}-\dfrac{\boldsymbol{x^2-2y^2}}{\boldsymbol{x^2-y^2}}$

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

Ex # 6.2

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$
$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$
$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$
$$=\frac{3y^2-2xy}{x^2-y^2}$$

(*v*) $\dfrac{\boldsymbol{x}}{\boldsymbol{2x^2+3xy+y^2}}-\dfrac{\boldsymbol{x-y}}{\boldsymbol{y^2-4x^2}}+\dfrac{\boldsymbol{y}}{\boldsymbol{2x^2+xy-y^2}}$

Solution:

$$\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$$
$$=\frac{x}{2x^2+2xy+1xy+y^2}-\frac{x-y}{-4x^2+y^2}+\frac{y}{2x^2+2xy-1xy-y^2}$$
$$=\frac{x}{2x(x+y)+y(x+y)}-\frac{x-y}{-(4x^2-y^2)}+\frac{y}{2x(x+y)-y(x+y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x)^2-y^2}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x+y)(2x-y)}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x(2x-y)+(x-y)(x+y)+y(2x+y)}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2-xy+x^2-y^2+2xy+y^2}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2+x^2-xy+2xy-y^2+y^2}{(x+y)((2x)^2-y^2)}$$
$$=\frac{3x^2+xy}{(x+y)(4x^2-y^2)}$$

(*vi*) $\dfrac{\boldsymbol{a}}{\boldsymbol{3x-y}}+\dfrac{\boldsymbol{a}}{\boldsymbol{3x+y}}-\dfrac{\boldsymbol{6ax}}{\boldsymbol{9x^2-y^2}}$

Solution:

$$\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$$
$$=\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{(3x)^2-y^2}$$

Chapter # 6

Ex # 6.2

$$= \frac{a}{3x-y} + \frac{a}{3x+y} - \frac{6ax}{(3x+y)(3x-y)}$$

$$= \frac{a(3x+y) + a(3x-y) - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{3ax + ay + 3ax - ay - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{3ax + 3ax - 6ax + ay - ay}{(3x+y)(3x-y)}$$

$$= \frac{6ax - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{0}{(3x+y)(3x-y)}$$

$$= 0$$

(vii) $\boldsymbol{\frac{y}{x-y} + \frac{y}{x+y} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}}$

Solution:

$$\frac{y}{x-y} + \frac{y}{x+y} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{y(x+y) + y(x-y)}{(x-y)(x+y)} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{xy + y^2 + xy - y^2}{(x-y)(x+y)} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{xy + xy + y^2 - y^2}{x^2-y^2} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2xy}{x^2-y^2} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2xy(x^2+y^2) + 2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2x^3y + 2xy^3 + 2x^3y - 2xy^3}{(x^2)^2 - (y^2)^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2x^3y + 2x^3y + 2xy^3 - 2xy^3}{x^4-y^4} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{4x^3y}{x^4-y^4} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{4x^3y(x^4+y^4) + 4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$$

Ex # 6.2

$$= \frac{4x^7y + 4x^3y^5 + 4x^7y - 4x^3y^5}{(x^4)^2 - (y^4)^2}$$

$$= \frac{4x^7y + 4x^7y + 4x^3y^5 - 4x^3y^5}{x^8-y^8}$$

$$= \frac{8x^7y}{x^8-y^8}$$

$(viii)$ $\boldsymbol{\frac{1}{a^2+7a+10} + \frac{1}{a^2+10a+16}}$

Solution:

$$\frac{1}{a^2+7a+10} + \frac{1}{a^2+10a+16}$$

$$= \frac{1}{a^2+2a+5a+10} + \frac{1}{a^2+2a+8a+16}$$

$$= \frac{1}{a(a+2)+5(a+2)} + \frac{1}{a(a+2)+8(a+2)}$$

$$= \frac{1}{(a+2)(a+5)} + \frac{1}{(a+2)(a+8)}$$

$$= \frac{1(a+8) + 1(a+5)}{(a+2)(a+5)(a+8)}$$

$$= \frac{a+8+a+5}{(a+2)(a+5)(a+8)}$$

$$= \frac{a+a+8+5}{(a+2)(a+5)(a+8)}$$

$$= \frac{2a+13}{(a+2)(a+5)(a+8)}$$

(ix) $\boldsymbol{\frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}}$

Solution:

$$\frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{1(a+b) + 1(a-b)}{(a-b)(a+b)} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{a+b+a-b}{(a-b)(a+b)} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{a+a+b-b}{a^2-b^2} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a}{a^2-b^2} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

Chapter # 6

Ex # 6.2

$$= \frac{2a(a^2+b^2)+2a(a^2-b^2)}{(a^2-b^2)(a^2+b^2)} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2ab^2+2a^3-2ab^2}{(a^2)^2-(b^2)^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2a^3+2ab^2-2ab^2}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3(a^4+b^4)+4a^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}$$

$$= \frac{4a^7+4a^3b^4+4a^7-4a^3b^4}{(a^4)^2-(b^4)^2}$$

$$= \frac{4a^7+4a^7+4a^3b^4-4a^3b^4}{a^8-b^8}$$

$$= \frac{8a^7}{a^8-b^8}$$

(x) $\dfrac{\boldsymbol{x^2-xy+y^2}}{\boldsymbol{x^3+y^3}} + \dfrac{\boldsymbol{x^2+xy+y^2}}{\boldsymbol{x^3-y^3}} - \dfrac{\boldsymbol{1}}{\boldsymbol{x^2-y^2}}$

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3} + \frac{x^2+xy+y^2}{x^3-y^3} - \frac{1}{x^2-y^2}$$

$$= \frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)} + \frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1}{x+y} + \frac{1}{x-y} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$= \frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$= \frac{x+x-y+y-1}{x^2-y^2}$$

$$= \frac{2x-1}{x^2-y^2}$$

Ex # 6.2

Q2: Simplify

(*i*) $\dfrac{\boldsymbol{x^2-25}}{\boldsymbol{5-x}}$

Solution:

$$\frac{x^2-25}{5-x}$$

$$= \frac{x^2-(5)^2}{-x+5}$$

$$= \frac{(x+5)(x-5)}{-(x-5)}$$

$$= -(x+5)$$

(*ii*) $\dfrac{\boldsymbol{x^2+5x+4}}{\boldsymbol{4y^3}} \times \dfrac{\boldsymbol{2y^2}}{\boldsymbol{x^2+3x+2}}$

Solution:

$$\frac{x^2+5x+4}{4y^3} \times \frac{2y^2}{x^2+3x+2}$$

$$= \frac{x^2+4x+1x+4}{4y.y.y} \times \frac{2y.y}{x^2+2x+1x+2}$$

$$= \frac{x(x+4)+1(x+4)}{2y} \times \frac{1}{x(x+2)+1(x+2)}$$

$$= \frac{(x+4)(x+1)}{2y} \times \frac{1}{(x+2)(x+1)}$$

$$= \frac{x+4}{2y} \times \frac{1}{x+2}$$

$$= \frac{x+4}{2y(x+2)}$$

(*iii*) $\dfrac{\boldsymbol{x^2-5x+4}}{\boldsymbol{x^3-3x-4}} \div \dfrac{\boldsymbol{x^3-4x^2+x-4}}{\boldsymbol{2x-1}}$

Solution:

$$\frac{x^2-5x+4}{x^2-3x-4} \div \frac{x^3-4x^2+x-4}{2x-1}$$

$$= \frac{x^2-5x+4}{x^2-3x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x^2-4x-1x+4}{x^2-4x+1x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x(x-4)-1(x-4)}{x(x-4)+1(x-4)} \times \frac{2x-1}{x^2(x-4)+1(x-4)}$$

$$= \frac{(x-4)(x-1)}{(x-4)(x+1)} \times \frac{2x-1}{(x-4)(x^2+1)}$$

Chapter # 6

Ex # 6.2

$$= \frac{(x-1)}{(x+1)} \times \frac{2x-1}{(x-4)(x^2+1)}$$

$$= \frac{(x-1)(2x-1)}{(x+1)(x-4)(x^2+1)}$$

(iv) $\boldsymbol{\frac{a(a+b)}{a^3-b^3} \times \frac{a^2+ab+b^2}{a^2+b^2}}$

Solution:

$$\frac{a(a+b)}{a^3-b^3} \times \frac{a^2+ab+b^2}{a^2+b^2}$$

$$= \frac{a(a+b)}{(a-b)(a^2+ab+b^2)} \times \frac{a^2+ab+b^2}{a^2+b^2}$$

$$= \frac{a(a+b)}{(a-b)} \times \frac{1}{a^2+b^2}$$

$$= \frac{a(a+b)}{(a-b)(a^2+b^2)}$$

(v) $\boldsymbol{\frac{7}{x^2-4} \div \frac{xy}{x+2}}$

Solution:

$$\frac{7}{x^2-4} \div \frac{xy}{x+2}$$

$$= \frac{7}{x^2-2^2} \times \frac{x+2}{xy}$$

$$= \frac{7}{(x+2)(x-2)} \times \frac{x+2}{xy}$$

$$= \frac{7}{x-2} \times \frac{1}{xy}$$

$$= \frac{7}{xy(x-2)}$$

(vi) $\boldsymbol{\frac{a^3-b^3}{a^4-b^4} \div \frac{a^2+ab+b^2}{a^2+b^2}}$

Solution:

$$\frac{a^3-b^3}{a^4-b^4} \div \frac{a^2+ab+b^2}{a^2+b^2}$$

$$= \frac{a^3-b^3}{a^4-b^4} \times \frac{a^2+b^2}{a^2+ab+b^2}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)} \times \frac{a^2+b^2}{a^2+ab+b^2}$$

Ex # 6.2

$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)} \times \frac{a^2+b^2}{a^2+ab+b^2}$$

$$= \frac{1}{(a+b)} \times \frac{1}{1}$$

$$= \frac{1}{(a+b)}$$

(vii) $\boldsymbol{\frac{2x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8}}$

Solution:

$$\frac{2x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8}$$

$$= \frac{2x}{3x-12} \times \frac{x^2-6x+8}{x^2-2x}$$

$$= \frac{2x}{3(x-4)} \times \frac{x^2-2x-4x+8}{x(x-2)}$$

$$= \frac{2x}{3(x-4)} \times \frac{x(x-2)-4(x-2)}{x(x-2)}$$

$$= \frac{2x}{3(x-4)} \times \frac{(x-2)(x-4)}{x(x-2)}$$

$$= \frac{2}{3} \times \frac{1}{1}$$

$$= \frac{2}{3}$$

$(viii)$ $\boldsymbol{\frac{a^4-8a}{2a^2+5a-3} \times \frac{2a-1}{a^2+2a+4} \div \frac{a^2-2a}{a+3}}$

Solution:

$$\frac{a^4-8a}{2a^2+5a-3} \times \frac{2a-1}{a^2+2a+4} \div \frac{a^2-2a}{a+3}$$

$$= \frac{a^4-8a}{2a^2+5a-3} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a^2-2a}$$

$$= \frac{a(a^3-8)}{2a^2+6a-1a-3} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a(a-2)}$$

$$= \frac{a(a^3-2^3)}{2a(a+3)-1(a+3)} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a(a-2)}$$

$$= \frac{a(a-2)(a^2+2a+4)}{(a+3)(2a-1)} \times \frac{2a-1}{a^2+2a+4} \times \frac{a+3}{a(a-2)}$$

$$= 1$$

Chapter # 6

Ex # 6.2

(ix) $\dfrac{9-x^2}{x^4+6x^3} \div \dfrac{x^3-2x^2-3x}{x^2+7x+6}$

Solution:

$$\frac{9-x^2}{x^4+6x^3} \div \frac{x^3-2x^2-3x}{x^2+7x+6}$$

$$= \frac{-x^2+9}{x^4+6x^3} \times \frac{x^2+7x+6}{x^3-2x^2-3x}$$

$$= \frac{-(x^2-9)}{x^3(x+6)} \times \frac{x^2+1x+6x+6}{x(x^2-2x-3)}$$

$$= \frac{-(x^2-3^2)}{x^3(x+6)} \times \frac{x(x+1)+6(x+1)}{x(x^2-3x+1x-3)}$$

$$= \frac{-(x+3)(x-3)}{x^3(x+6)} \times \frac{(x+1)(x+6)}{x[x(x-3)+1(x-3)]}$$

$$= \frac{-(x+3)(x-3)}{x^3(x+6)} \times \frac{(x+1)(x+6)}{x[(x-3)(x+1)]}$$

$$= \frac{-(x+3)}{x^3} \times \frac{1}{x}$$

$$= \frac{-(x+3)}{x^4}$$

(x) $\dfrac{ax+ab+cx+bc}{a^2-x^2} \times \dfrac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

Solution:

$$\frac{ax+ab+cx+bc}{a^2-x^2} \times \frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$$

$$= \frac{ax+ab+cx+bc}{-x^2+a^2} \times \frac{x^2-2ax+a^2}{x^2+bx+ax+ab}$$

$$= \frac{a(x+b)+c(x+b)}{-(x^2-a^2)} \times \frac{(x-a)^2}{x(x+b)+a(x+b)}$$

$$= -\frac{(x+b)(a+c)}{(x+a)(x-a)} \times \frac{(x-a)(x-a)}{(x+b)(x+a)}$$

$$= -\frac{(a+c)}{(x+a)} \times \frac{(x-a)}{(x+a)}$$

$$= -\frac{(a+c)(x-a)}{(x+a)^2}$$

Ex # 6.3

Square root

Square root of a number is a number that can be multiplied by itself to produce the original

Square root of an algebraic expression can be found out by the following two methods.

(i) Factorization Method

(ii) Division Method

Square root by Factorization

In this method make the expression a perfect square then finds square root.

Example # 20

Find the square root of $\boldsymbol{x^2+ax+\frac{1}{4}a^2}$ **by factorization**

Solution:

$$x^2+ax+\frac{1}{4}a^2$$

$$x^2+ax+\frac{1}{4}a^2 = (x)^2+2(x)\left(\frac{1}{2}a\right)+\left(\frac{1}{2}a\right)^2$$

$$x^2+ax+\frac{1}{4}a^2 = \left(x+\frac{1}{2}a\right)^2$$

Now take square root on B.S

$$\sqrt{x^2+ax+\frac{1}{4}a^2} = \sqrt{\left(x+\frac{1}{2}a\right)^2}$$

$$\sqrt{x^2+ax+\frac{1}{4}a^2} = \pm\left(x+\frac{1}{2}a\right)$$

Example # 21

Find the square root of $\boldsymbol{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}$

Solution:

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+25+2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = x^2+\frac{1}{x^2}+2-10\left(x+\frac{1}{x}\right)+25$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = \left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(5)+(5)^2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = \left(x+\frac{1}{x}-5\right)^2$$

Chapter # 6

Ex # 6.3

Taking square root on B.S

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left(x+\frac{1}{x}-5\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\pm\left(x+\frac{1}{x}-5\right)$$

Square root by Division

طریقہ:

Expression کو Descending ترتیب میں لکھیں۔

پہلے expression کا Square root لینگے پھر Divisor اور Quotient میں لکھیں گے۔

Divisor اور Quotient کو آپس میں Multiply کریں اور پہلے expression کے نیچے لکھیں پھر Subtract کریں تو Remainder حاصل ہو جائے گا

Divisor کو ڈبل کر دے اور Remainder کو اس پر Divide کر دے اور جو Term آئے گا تو Divisor اور Quotient میں اس کو لکھیں۔

اب اس Quotient کو پورے Divisor کے ساتھ Multiply کرے پھر Subtract کرے

اب Divisor کے دوسرے Term کو ڈبل کرے اور اوپر کا طریقہ دوبارہ کریں۔

Find the square root of $16x^4-24x^3+25x^2-12x+4$

Solution:

Write the expression in descending order

$16x^4-24x^3+25x^2-12x+4$

Take the square root of first element of expression.

$$\sqrt{16x^4}=4x^2$$

Write $4x^2$ in divisor and quotient

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4
\end{array}$$

Multiply the divisor and quotient and write it under first expression then subtract from given expression to get the remainder.

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
 & -24x^3+25x^2-12x+4
\end{array}$$

Now twice the divisor

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2 & -24x^3+25x^2-12x+4
\end{array}$$

Divide the 2nd expression by this divisor then write that term in quotient and with this divisor.

$$\frac{-24x^3}{8x^2}=-3x$$

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4
\end{array}$$

Multiply this quotient with entire divisor

$$-3x(8x^2-3x)=-24x^3+9x^2$$

Write $-24x^3+9x^2$ under given expression then subtract it.

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
 & 16x^2-12x+4
\end{array}$$

Now twice the 2nd term of the divisor

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
8x^2-6x & 16x^2-12x+4
\end{array}$$

Repeat the above procedure.

Divide $16x^2$ by divisor $8x^2$ then write that term in quotient and with this divisor.

$$\frac{16x^2}{8x^2}=2$$

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x+2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
8x^2-6x+2 & 16x^2-12x+4
\end{array}$$

Multiply this quotient with entire divisor

$$2(8x^2-6x+2)=16x^2-12x+4$$

Write $16x^2-12x+4$ under given expression then subtract it.

$$\begin{array}{r|l}
\multicolumn{2}{c}{4x^2-3x+2} \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp24x^3\pm9x^2 \\ \cline{2-2}
8x^2-6x+2 & 16x^2-12x+4 \\
 & \pm16x^2\mp12x\pm4 \\ \cline{2-2}
 & 0
\end{array}$$

Chapter # 6

Ex # 6.3

Example # 22

Find the square root of $16x^4 - 24x^3 + 25x^2 - 12x + 4$

Solution:

Now

$$\begin{array}{l|l} & 4x^2 - 3x + 2 \\ \hline 4x^2 & 16x^4 - 24x^3 + 25x^2 - 12x + 4 \\ & \pm 16x^4 \\ \hline 8x^2 - 3x & -24x^3 + 25x^2 - 12x + 4 \\ & \mp 24x^3 \pm 9x^2 \\ \hline 8x^2 - 6x + 2 & 16x^2 - 12x + 4 \\ & \pm 16x^2 \mp 12x \pm 4 \\ \hline & 0 \end{array}$$

So

$$\sqrt{16x^4 - 24x^3 + 25x^2 - 12x + 4} = \pm(4x^2 - 3x + 2)$$

Example # 20

Find the square root of $\frac{x^2}{4} + 4x^2 + \frac{ax^2}{3} + \frac{a^2}{9} - 2x^3 - \frac{4ax}{3}$

Solution:

$$\frac{x^2}{4} + 4x^2 + \frac{ax^2}{3} + \frac{a^2}{9} - 2x^3 - \frac{4ax}{3}$$

The descending order of the expression are:

$$\frac{x^2}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9}$$

Now

$$\begin{array}{l|l} & \frac{x^2}{2} - 2x + \frac{a}{3} \\ \hline \frac{x^2}{2} & \frac{x^4}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9} \\ & \pm\frac{x^4}{4} \\ \hline x^2 - 2x & -2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9} \\ & \mp 2x^3 \pm 4x^2 \\ \hline x^2 - 4x + \frac{a}{3} & \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9} \\ & \pm\frac{ax^2}{3} \mp \frac{4ax}{3}x \pm \frac{a^2}{9} \\ \hline & 0 \end{array}$$

So

$$\sqrt{\frac{x^4}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9}} = \pm\left(\frac{x^2}{2} - 2x + \frac{a}{3}\right)$$

Ex # 6.3

Example # 24

What should be added to

What should be subtracted from

For what value of x

The expression $9x^4 - 12x^3 + 10x^2 - 3x - 3$

to make the perfect square

Solution:

$$9x^4 - 12x^3 + 10x^2 - 3x - 3$$

$$\begin{array}{l|l} & 3x^2 - 2x + 1 \\ \hline 3x^2 & 9x^4 - 12x^3 + 10x^2 - 3x - 3 \\ & \pm 9x^4 \\ \hline 6x^2 - 2x & -12x^3 + 10x^2 - 3x - 3 \\ & \mp 12x^3 \pm 4x^2 \\ \hline 6x^2 - 4x + 1 & 6x^2 - 3x - 3 \\ & \pm 6x^2 \mp 4x \pm 1 \\ \hline & x - 4 \end{array}$$

As for perfect square, Remainder $= 0$

$-x + 4$ should be Added to $9x^4 - 12x^3 + 10x^2 - 3x - 3$ will become perfect square.

$-x + 4 + (x - 4) = -x + 4 + x - 4$

$-x + 4 + (x - 4) = 0$

$x - 4$ should be Subtracted to $9x^4 - 12x^3 + 10x^2 - 3x - 3$ will become perfect square.

$x - 4 - (x - 4) = x - 4 - x + 4$

$x - 4 - (x - 4) = 0$

For x

$x - 4 = 0$

$x = 4$

Chapter # 6

Exercise# 6.3

Page # 169

Q1: Find the square root by factorization method.

$(i)\ \boldsymbol{x^2+4x+4}$

Solution:

x^2+4x+4

$x^2+4x+4 = x^2+2(x)(2)+2^2$

$x^2+4x+4 = (x+2)^2$

Taking Square on B.S

$\sqrt{x^2+4x+4} = \pm\sqrt{(x+2)^2}$

$\sqrt{x^2+4x+4} = \pm(x+2)$

$(ii)\ \boldsymbol{(x-y)^2+6(x-y)+9}$

Solution:

$(x-y)^2+6(x-y)+9$

$(x-y)^2+6(x-y)+9 = (x-y)^2+2(x-y)(3)+3^2$

$(x-y)^2+6(x-y)+9 = (x-y+3)^2$

Taking Square on B.S

$\sqrt{(x-y)^2+6(x-y)+9} = \pm\sqrt{(x-y+3)^2}$

$\sqrt{(x-y)^2+6(x-y)+9} = \pm(x-y+3)$

$(iii)\ \boldsymbol{x^2y^2-8xy+16}$

Solution:

$x^2y^2-8xy+16$

$x^2y^2-8xy+16 = (xy)^2+2(xy)(4)+4^2$

$x^2y^2-8xy+16 = (xy+4)^2$

Taking Square on B.S

$\sqrt{x^2y^2-8xy+16} = \pm\sqrt{(xy+4)^2}$

$\sqrt{x^2y^2-8xy+16} = \pm(xy+4)$

$(iv)\ \boldsymbol{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}$

Solution:

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18$$

$$= x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+2+16$$

$$= x^2+\frac{1}{x^2}+2-8\left(x+\frac{1}{x}\right)+16$$

$$= \left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(4)+(4)^2$$

$$= \left(x-\frac{1}{x}+4\right)^2$$

Ex # 6.3

Now

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18 = \left(x-\frac{1}{x}+4\right)^2$$

Taking Square on B.S

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18} = \pm\sqrt{\left(x-\frac{1}{x}+4\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18} = \pm\left(x-\frac{1}{x}+4\right)$$

$(v)\ \boldsymbol{(x+1)(x+2)(x+3)+1}$

Solution:

$x(x+1)(x+2)(x+3)+1$

Rearranging accordingly $0+3=1+2$

$= x(x+3)(x+1)(x+2)+1$

$= (x^2+3x)(x^2+2x+1x+2)+1$

$= (x^2+3x)(x^2+3x+2)+1$

$Let\ \ x^2+3x = y$

$= y^2+2y+1$

$= (y)^2+2(y)(1)+(1)^2$

$= (y+1)^2$

But $y = x^2+3x$

$= (x^2+3x+1)^2$

Now

$x(x+1)(x+2)(x+3)+1 = (x^2+3x+1)^2$

Taking Square on B.S

$\sqrt{x(x+1)(x+2)(x+3)+1} = \pm\sqrt{(x^2+3x+1)^2}$

$\sqrt{x(x+1)(x+2)(x+3)+1} = \pm(x^2+3x+1)$

Chapter # 6

Ex # 6.3

(vi) $\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$

Solution:

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=x^2+\frac{1}{x^2}+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

Subtract and Add 2

$$=x^2+\frac{1}{x^2}-2+2+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2+4-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}+4$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9+16}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{25}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2$$

$$=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Now

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\sqrt{\left(x-\frac{1}{x}-\frac{5}{2}\right)^2}$$

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\left(x-\frac{1}{x}-\frac{5}{2}\right)$$

(vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

Solution:

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}+2\right)+12$$

Ex # 6.3

$$=\left(x^2+\frac{1}{x^2}\right)^2-4x^2-\frac{4}{x^2}-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(2)+(4)^2$$

$$=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Now

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\sqrt{\left(x^2+\frac{1}{x^2}-2\right)^2}$$

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

(viii) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}$

Solution:

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}$$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}$$

$$=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Now

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Taking square root on B.S

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\sqrt{\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2}$$

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

Chapter # 6

Ex # 6.3

Q2: Find the square root of the following by Division method.

(i) $\mathbf{4x^4 - 4x^3 + 13x^2 - 6x + 9}$

Solution:

$$4x^4 - 4x^3 + 13x^2 - 6x + 9$$

$$\begin{array}{r|l}
 & 2x^2 - x + 3 \\ \hline
2x^2 & 4x^4 - 4x^3 + 13x^2 - 6x + 9 \\
 & \pm 4x^4 \\ \hline
4x^2 - x & -4x^3 + 13x^2 - 6x + 9 \\
 & \mp 4x^3 \pm x^2 \\ \hline
4x^2 - 2x + 3 & 12x^2 - 6x + 9 \\
 & \pm 12x^2 \mp 6x \pm 9 \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{4x^4 - 4x^3 + 13x^2 - 6x + 9} = \pm(2x^2 - x + 3)$$

(ii) $\mathbf{x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16}$

Solution:

$$x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16$$

$$\begin{array}{r|l}
 & x^2 + \frac{x}{2} - 4 \\ \hline
x^2 & x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16 \\
 & \pm x^4 \\ \hline
2x^2 + \frac{x}{2} & x^3 - \frac{31}{4}x^2 - 4x + 16 \\
 & \pm x^3 \pm \frac{x^2}{4} \\ \hline
2x^2 + x - 4 & -8x^2 - 4x + 16 \\
 & \mp 8x^2 \mp 4x \pm 16 \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16} = \pm\left(x^2 + \frac{x}{2} - 4\right)$$

(iii) $\mathbf{x^2 - 2x + 1 + 2xy - 2y + y^2}$

Solution:

$$x^2 - 2x + 1 + 2xy - 2y + y^2$$

Ex # 6.3

$$\begin{array}{r|l}
 & x - 1 + y \\ \hline
x & x^2 - 2x + 1 + 2xy - 2y + y^2 \\
 & \pm x^2 \\ \hline
2x - 1 & -2x + 1 + 2xy - 2y + y^2 \\
 & \mp 2x \pm 1 \\ \hline
2x - 2 + y & 2xy - 2y + y^2 \\
 & \pm 2xy \mp 2y \pm y^2 \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{x^2 - 2x + 1 + 2xy - 2y + y^2} = \pm(x - 1 + y)$$

(iv) $\mathbf{\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36}$

Solution:

$$\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36$$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2(x^2)\left(\frac{1}{x^2}\right) - 12x^2 + \frac{12}{x^2} + 36$$

$$= x^4 + \frac{1}{x^4} - 2 - 12x^2 + \frac{12}{x^2} + 36$$

Arrange it in ascending order

$$= x^4 - 12x^2 - 2 + 36 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$= x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$\begin{array}{r|l}
 & x^2 - 6 - \frac{1}{x^2} \\ \hline
x^2 & x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\
 & \pm x^4 \\ \hline
2x^2 - 6 & -12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\
 & \mp 12x^2 \pm 36 \\ \hline
2x^2 - 12 - \frac{1}{x^2} & -2 + \frac{12}{x^2} + \frac{1}{x^4} \\
 & \mp 2 \pm \frac{12}{x^2} \pm \frac{1}{x^4} \\ \hline
 & 0
\end{array}$$

So

$$\sqrt{\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36} = \pm\left(x^2 - 6 - \frac{1}{x^2}\right)$$

Chapter # 6

Ex # 6.3

Q3 (i): For what value of k the expression
$$\mathbf{4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}}$$
will become perfect square.

Solution:

$4x^4+32x^2+96+\dfrac{128}{x^2}+\dfrac{k}{x^4}$

$$\begin{array}{r|l}
 & 2x^2+8+\frac{8}{x^2} \\ \hline
2x^2 & 4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4} \\
 & \pm 4x^4 \\ \hline
4x^2+8 & 32x^2+96+\frac{128}{x^2}+\frac{k}{x^4} \\
 & \pm 32x^2 \pm 64 \\ \hline
4x^2+16+\frac{8}{x^2} & 32+\frac{128}{x^2}+\frac{k}{x^4} \\
 & \pm 32 \pm \frac{128}{x^2} \pm \frac{64}{x^4} \\ \hline
 & \frac{k}{x^4}-\frac{64}{x^4}
\end{array}$$

As for perfect square, Remainder $=0$

$\dfrac{k}{x^4}-\dfrac{64}{x^4}=0$

$\dfrac{k-64}{x^4}=0$

$k-64=0\times x^4$

$k-64=0$

$k=64$

Q3 (ii):

(i) What should be added to

(ii) What should be subtracted to

(iii) For what value of x the expression

$\mathbf{4x^4-12x^3+17x^2-13x+6}$ **so that it becomes perfect square**

Solution:

$4x^4-12x^3+17x^2-13x+6$

$$\begin{array}{r|l}
 & 2x^2-3x+2 \\ \hline
2x^2 & 4x^4-12x^3+17x^2-13x+6 \\
 & \pm 4x^4 \\ \hline
4x^2-3x & -12x^3+17x^2-13x+6 \\
 & \mp 12x^3 \pm 9x^2 \\ \hline
4x^2-6x+2 & 8x^2-13x+6 \\
 & \pm 8x^2 \mp 12x \pm 4 \\ \hline
 & -x+2
\end{array}$$

As for perfect square, Remainder $=0$

Ex # 6.3

$x-2$ should be Added to $4x^4-12x^3+17x^2-13x+6$ will become perfect square.

$-x+2+(x-2)=-x+2+x-2$

$-x+2+(x-2)=0$

$-x+2$ should be Subtracted to $4x^4-12x^3+17x^2-13x+6$ will become perfect square.

$-x+2-(-x+2)=-x+2+x-2$

$-x+2-(-x+2)=0$

For x

$-x+2=0$

$-x=-2$

$x=2$

Q4: What should be subtracted and added to the expression $x^4-4x^3+10x+7$ so that the expression is made perfect square?

Solution:

$x^4-4x^3+10x+7$

$$\begin{array}{r|l}
 & x^2-2x-2 \\ \hline
x^2 & x^4-4x^3+10x+7 \\
 & \pm x^4 \\ \hline
2x^2-2x & -4x^3+10x+7 \\
 & \mp 4x^3 \quad \pm 4x^2 \\ \hline
2x^2-4x-2 & -4x^2+10x+7 \\
 & \mp 4x^2 \pm 8x \pm 4 \\ \hline
 & 2x+3
\end{array}$$

As for perfect square, Remainder $=0$

$-2x-3$ should be Added to $x^4-4x^3+10x+7$ will become perfect square.

$-2x-3+(2x+3)=2x+3-2x-3$

$-2x-3+(2x+3)=0$

$2x+3$ should be Subtracted to $x^4-4x^3+10x+7$ will become perfect square.

$2x+3-(2x+3)=2x+3-2x-3$

$2x+3-(2x+3)=0$

Chapter # 6

Ex # 6.3

Q5 (i): Find the value of l and m for which expression will become perfect square

$\boldsymbol{x^4 + 4x^3 + 16x^2 + lx + m}$

Solution:

$x^4 + 4x^3 + 16x^2 + lx + m$

	$x^2 + 2x + 6$
x^2	$x^4 + 4x^3 + 16x^2 + lx + m$
	$\pm x^4$
$2x^2 + 2x$	$4x^3 + 16x^2 + lx + m$
	$\pm 4x^3 \pm 4x^2$
$2x^2 + 4x + 6$	$12x^2 + lx + m$
	$\pm 12x^2 \pm 24x \pm 36$
	$lx - 24x + m - 36$

As for perfect square, Remainder $= 0$

$lx - 24x + m - 36 = 0$

$(l - 24)x + (m - 36) = 0$

This $(l - 24)x + (m - 36) = 0$ when

$(l - 24)x + (m - 36) = 0x + 0$

By compare the co-efficient of x and constant

$$l - 24 = 0$$
$$l = 24$$

And $m - 36 = 0$

$$m = 36$$

Hence

$$l = 24 \text{ and } m = 36$$

Q5 (ii): Find the value of l and m for which expression will become perfect square

$\boldsymbol{49x^4 - 70x^3 + 109x^2 + lx - m}$

Solution:

$49x^4 - 70x^3 + 109x^2 + lx - m$

	$7x^2 - 5x + 6$
$7x^2$	$49x^4 - 70x^3 + 109x^2 + lx - m$
	$\pm 49x^4$
$14x^2 - 5x$	$-70x^3 + 109x^2 + lx - m$
	$\mp 70x^3 \pm 25x^2$
$14x^2 - 10x + 6$	$84x^2 + lx - m$
	$\pm 84x^2 \mp 60x \pm 36$
	$lx + 60x - m - 36$

As for perfect square, Remainder $= 0$

$lx + 60x - m - 36 = 0$

Ex # 6.3

$(l + 60)x + (-m - 36) = 0$

This $(l + 60)x + (-m - 36) = 0$ when

$(l + 60)x + (-m - 36) = 0x + 0$

By compare the co-efficient of x and constant

$$l + 60 = 0$$
$$l = -60$$

And $-m - 36 = 0$

$$-m = 36$$
$$m = -36$$

Hence

$$l = -60 \text{ and } m = -36$$

Review Exercise # 6

Page # 171

Q2: Simplify the following.

$(\mathbf{i}): \ \dfrac{\mathbf{5}}{\mathbf{2s+4}} - \dfrac{\mathbf{3}}{\mathbf{s^2+3s+2}} + \dfrac{\boldsymbol{s}}{\mathbf{s^2-s-2}}$

Solution:

$$\frac{5}{2s+4} - \frac{3}{s^2+3s+2} + \frac{s}{s^2-s-2}$$

$$= \frac{5}{2(s+2)} - \frac{3}{s^2+2s+1s+2} + \frac{s}{s^2-2s+1s-2}$$

$$= \frac{5}{2(s+2)} - \frac{3}{s(s+2)+1(s+2)} + \frac{s}{s(s-2)+1(s-2)}$$

$$= \frac{5}{2(s+2)} - \frac{3}{(s+2)(s+1)} + \frac{s}{(s-2)(s+1)}$$

$$= \frac{5(s+1)(s-2) - 3 \times 2(s-2) + s \times 2(s+2)}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5(s^2-2s+1s-2) - 6(s-2) + 2s(s+2)}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5(s^2-1s-2) - 6s + 12 + 2s^2 + 4s}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5s^2 - 5s - 10 - 6s + 12 + 2s^2 + 4s}{2(s+2)(s+1)(s-2)}$$

$$= \frac{5s^2 + 2s^2 - 5s - 6s + 4s - 10 + 12}{2(s+2)(s+1)(s-2)}$$

$$= \frac{7s^2 - 11s + 4s - 2}{2(s+2)(s+1)(s-2)}$$

$$= \frac{7s^2 - 7s - 2}{2(s+2)(s+1)(s-2)}$$

Chapter # 6

Review Ex # 6

$(ii).\dfrac{\boldsymbol{a}}{(\boldsymbol{c}-\boldsymbol{a})(\boldsymbol{a}-\boldsymbol{b})}+\dfrac{\boldsymbol{b}}{(\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{b}-\boldsymbol{c})}+\dfrac{\boldsymbol{c}}{(\boldsymbol{b}-\boldsymbol{c})(\boldsymbol{c}-\boldsymbol{a})}$

Solution:

$$\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$$

$$=\frac{a(b-c)+b(c-a)+c(a-b)}{(c-a)(a-b)(b-c)}$$

$$=\frac{ab-ac+bc-ab+ac-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{ab-ab-ac+ac+bc-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{0}{(a-b)(b-c)(c-a)}$$

$$=0$$

$(\mathbf{iii}):\ \dfrac{\boldsymbol{x^2-4}}{\boldsymbol{xy^2}}\cdot\dfrac{\boldsymbol{2xy}}{\boldsymbol{x^2-4x+4}}$

Solution:

$$\frac{x^2-4}{xy^2}\cdot\frac{2xy}{x^2-4x+4}$$

$$=\frac{x^2-2^2}{xyy}\cdot\frac{2xy}{x^2-2(x)(2)+2^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)(x+2)}$$

$$=\frac{(x-2)}{y}\cdot\frac{2}{(x+2)}$$

$$=\frac{2(x-2)}{y(x+2)}$$

$(\mathbf{iv}):\ \dfrac{\boldsymbol{a^3-b^3}}{\boldsymbol{a^4-b^4}}\div\dfrac{\boldsymbol{a^2+ab+b^2}}{\boldsymbol{a^2+b^2}}$

Solution:

$$\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a^3-b^3}{a^4-b^4}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

Review Ex # 6

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{1}{a+b}\times\frac{1}{1}$$

$$=\frac{1}{a+b}$$

Chapter # 6

Review Ex # 6

Q3: **Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3**

$x^3-6x^2+11x-6$ and x^3-4x+3

Solution:

$Let\ A = x^3-6x^2+11x-6$

$and\ B = x^3-4x+3$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F} \text{ --}equ\ (i)$$

First we find H.C.F

$x^3-4x+3 \;\big)\; x^3-6x^2+11x-6 \;\big(\; 1$

$\pm x^3 \qquad \mp 4x \pm 3$

$-3 \;\big)\; -6x^2+15x-9$

$2x^2-5x+3 \;\big)\; x^3-4x+3 \;\big(\; x+5$

$\times 2$

$2x^3-8x+6$

$\pm 2x^3 \pm 3x \qquad \mp 5x^2$

$5x^2-11x+6$

$\times 2$

$10x^2-22x+12$

$\pm 10x^2 \mp 25x \pm 15$

$3 \;\big)\; 3x-3$

$x-1 \;\big)\; 2x^2-5x+3 \;\big(\; 2x-3$

$\pm 2x^2 \mp 2x$

$-3x+3$

$\mp 3x \pm 3$

$\times$

$H.C.F = x-1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3-6x^2+11x-6)(x^3-4x+3)}{x-1}$$

Now by Simple Division

x^2-5x+6

$x-1 \;\big)\; x^3-6x^2+11x-6$

$\pm x^3 \mp x^2$

$-5x^2+11x-6$

$\mp 5x^2 \pm 5x$

$6x-6$

$\pm 6x \mp 6$

$\times$

$So\ L.C.M = (x^2-5x+6)(x^3-4x+3)$

Chapter # 6

Review Ex # 6

Q4: Find the square root of :

(i): $4x^2 - 12x + 9$

Solution:

$4x^2 - 12x + 9$

$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + (3)^2$

$4x^2 - 12x + 9 = (2x - 3)^2$

Taking Square on B.S

$\sqrt{4x^2 - 12x + 9} = \pm\sqrt{(2x - 3)^2}$

$\sqrt{4x^2 - 12x + 9} = \pm(2x - 3)$

Review Ex # 6

(ii): $x^4 + 4x^3 + 6x^2 + 4x + 1$

Solution:

$x^4 + 4x^3 + 6x^2 + 4x + 1$

	$x^2 + 2x + 1$
x^2	$x^4 + 4x^3 + 6x^2 + 4x + 1$
	$\pm x^4$
$2x^2 + 2x$	$4x^3 + 6x^2 + 4x + 1$
	$\pm 4x^3 \pm 4x^2$
$2x^2 + 4x + 1$	$2x^2 + 4x + 1$
	$\pm 2x^2 \pm 4x \pm 1$
	0

So

$$\sqrt{x^4 + 4x^3 + 6x^2 + 4x + 1} = \pm(x^2 + 2x + 1)$$

Think

$Q5$: Simplify $\dfrac{x^3 - y^3}{x^3 - z^3} \times \dfrac{x^2 + xy + xz + yz}{x^4 + x^2y^2 + y^4} \times \dfrac{x^3 + y^3}{x^2 - y^2}$

Solution:

$$\frac{x^3 - y^3}{x^3 + z^3} \times \frac{x^2 + xy + xz + yz}{x^4 + x^2y^2 + y^4} \times \frac{x^3 + y^3}{x^2 - y^2}$$

$$= \frac{(x - y)(x^2 + xy + y^2)}{(x + z)(x^2 - xz + z^2)} \times \frac{x(x + y) + z(x + y)}{x^4 + y^4 + x^2y^2} \times \frac{(x + y)(x^2 - xy + y^2)}{(x + y)(x - y)}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)}{(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2 + x^2y^2} \times \frac{(x^2 - xy + y^2)}{1}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - x^2y^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - (xy)^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x + y)(x^2 - xy + y^2)}{(x^2 + y^2 + xy)(x^2 + y^2 - xy)}$$

$$= \frac{1}{(x^2 - xz + z^2)} \times \frac{(x + y)}{1}$$

$$= \frac{(x + y)}{(x - z)(x^2 + xz + z^2)}$$